

A Simple Derivation of Lorentz Transformation based on Length Contraction

Using Galilean transformation x' as measured by an observer O' moving at constant velocity relative to stationary (relative to Earth) observer O is:

$$x' = x - vt \quad [1]$$

According to the stationary observer O' , X' appears contracted and, using the well known Lorentz-Fitzgerald contraction, it is:

$$x' \sqrt{1 - \frac{v^2}{c^2}} = x - vt \quad [2]$$

If we solve for x' we get the Lorentz transformation form X' to X and vice versa.

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [3]$$

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [4]$$

We can solve for t' and/or t using equations [3] and [4] and obtain the Lorentz transformation for time (t and/or t')

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [5]$$

The inverse transformation for [5] is;

$$t = \frac{t' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad [6]$$

Since the relative motion is restricted to x , $y'=y$ and $z'=z$. This concludes the derivation for Lorentz transformation. There a variety of derivations based on the two postulates of special relativity that requires more algebraic work.

