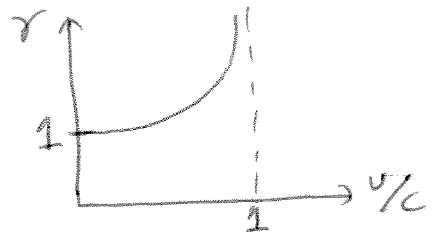
PHYS 301 Test #1 Name: Key



Time Dilation	Length Contraction
$\Delta t = \gamma \Delta t_0$	$L_0 = \gamma L$
$\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	

1. Sketch γ as a function of v/c, in the space above.
2. The mean lifetime of stationary muons is measured to be 2.2000 μs. The mean lifetime of high-speed muons in a burst of cosmic rays observed from Earth is measured to be 16.000 μs. To five significant figures, what is the speed parameter β of these cosmic-ray muons relative to Earth?

$$\Delta t = \gamma \Delta t_0$$
$$16 = 2.2\gamma \rightarrow \gamma = \frac{16}{2.2} = \frac{1}{\sqrt{1-\beta^2}}$$
$$1-\beta^2 = 0.018906$$
$$\beta^2 = 0.98109375 \rightarrow \boxed{\beta = 0.99050}$$

3. A 520 m long (measured when the spaceship is stationary) spaceship passes by the Earth. What length would the people on Earth say the spaceship was as it passed the Earth at 0.87c?

$$L_0 = \gamma L \rightarrow L = \frac{L_0}{\gamma} = \frac{520}{1/\sqrt{1-0.87^2}} =$$
$$L = 520 \times \sqrt{1-0.87^2} = 256\text{m}$$
$$\boxed{L = 256\text{m}}$$

4. A space traveler takes off from Earth and moves at speed 0.9900c toward the star Vega, which is 26.00 ly distant. How much time will have elapsed by Earth clocks (a) when the traveler reaches Vega and (b) when Earth observers receive word from the traveler that she has arrived? (c) How much older will Earth observers calculate the traveler to be when she reaches Vega than she was when she started the trip?

0.99c Vega 26LY E

$$\gamma = \frac{1}{\sqrt{1-0.99^2}} = 7.089$$

(a) $\text{time} = \frac{d}{v}$

$\text{time} = \frac{26\text{LY}}{0.99\text{c}}$

$\boxed{\text{time} = 26.26\text{y}}$

(b) time for signal $= 26\text{y}$

total time: $26.26 + 26$

$\boxed{\text{Tot. tim} = 52.26\text{y}}$

(c) $\Delta t = \gamma \Delta t_0$

$26.26 = 7.089 \Delta t_0$

$\frac{26.26}{7.089} = \Delta t_0 = 3.705\text{y}$

$\boxed{\Delta t_0 = 3.705\text{y}}$

5. An airplane whose rest length is 40.0 m is moving at uniform velocity with respect to Earth, at a speed of 930 m/s. By what fraction of its rest length is it shortened to an observer on Earth? [$(1+x)^n \approx 1+nx$, when x is small]

$$\frac{\Delta L}{L_0} = \frac{L_0 - L}{L_0} = 1 - \frac{L}{L_0} = 1 - \frac{1}{\gamma} = 1 - \left(1 - \frac{v^2}{c^2}\right)^{1/2}$$
$$= 1 - \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)$$
$$= 1 - 1 + \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{v^2}{c^2}$$
$$\frac{\Delta L}{L_0} = \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\left(\frac{930}{3\times10^8}\right)^2 = 4.805\times10^{-12}$$

$E = mc^2 = \gamma m_0 c^2 \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \beta = \frac{v}{c} \quad E = m_0 c^2 + K \quad E^2 = p^2c^2 + m_0^2 c^4$

6. The mass of an electron is 9.109 381 88 × 10^{-31} kg. Calculate the rest energy of the electron in MeV. [c = 2.99792458 x10^8 m/s, 1eV = 1.60217662 x 10^{-19} J]

$$E = M_0 c^2 = 9.10938188 \times 10^{-31} \times (2.99792458 \times 10^8)^2 \ J$$

$$= \frac{9.10938188 \times 10^{-31} \times (2.99792458 \times 10^8)^2}{1.60217662 \times 10^{-19}} \times \frac{1\ \text{Mev}}{10^6}$$

$$\boxed{E = 0.511\ \text{MeV}}$$

7. An electron moves with a speed of 0.6c. Find the total energy, kinetic energy, and momentum of the electron.

$$E = mc^2 = \gamma M_0 c^2 \qquad \gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.6^2}} = 1.25$$

$$E = 1.25 \times 0.511\ \text{Mev}$$

$$\boxed{E = 0.63875\ \text{MeV} = 1.023 \times 10^{-13}\ J}$$

$$K = E - M_0 c^2 = (0.63875 - 0.511)\ \text{Mev} = 0.12775\ \text{Mev}$$

$$\boxed{K = 0.12775\ \text{Mev} = 2.05 \times 10^{-14}\ J}$$

$$p^2 = \frac{E^2}{c^2} - m_0^2 c^2$$

$$p = \sqrt{\frac{E^2}{c^2} - m_0^2 c^2} = \sqrt{\frac{(1.023 \times 10^{-13})^2}{(3 \times 10^8)^2} - (9.11 \times 10^{-31})^2 \cdot (3 \times 10^8)^2} = \boxed{2.05 \times 10^{-22}\ \text{kg} \cdot \text{m/s}}$$

8. Quasars are thought to be the nuclei of active galaxies in the early stages of their formation. A typical quasar radiates energy at the rate of 10^{35} W. At what rate is the mass of this quasar being reduced to supply this energy? Express your answer in solar mass units per year, where one solar mass unit (1 smu = 2.0 × 10^{30} kg) is the mass of our Sun.

$$E = mc^2$$

$$\frac{dE}{dt} = \frac{dm}{dt} c^2 \rightarrow \frac{dm}{dt} = \frac{dE/dt}{c^2} = \frac{10^{35}}{(3 \times 10^8)^2} = 0.1111 \times 10^{19} = 1.111 \times 10^{18}\ \text{kg/s}$$

$$\boxed{\frac{dM}{dt} = 1.111 \times 10^{18}\ \text{kg/s}}$$

$$\frac{dm}{dt} = 1.111 \times 10^{18}\ \frac{\text{kg}}{\text{s}} \times \frac{M_\odot \times}{2 \times 10^{30}\ \text{kg}} \times \frac{365 \times 24 \times 3600\ \text{s}}{\text{year}}$$

$$\boxed{\frac{dm}{dt} = 1.75 \times 10^{-5}\ \text{SM/year}}$$

9. A particle's rest mass, m_0, its momentum magnitude, p, and its kinetic energy, K, are related by:

$$m_0 = \frac{(pc)^2 - K^2}{2Kc^2}.$$

For low particle speeds, show that the right side of the equation reduces to m.

$$M_0 = \frac{p^2c^2 - K^2}{2Kc^2}$$

$$= \frac{p^2 - (K^2/c^2)}{2K} \quad \text{small}$$

$$= \frac{p^2}{2K} = \frac{m^2v^2}{2\cdot\frac{1}{2}mv^2} = m$$

10. The Doppler shifted frequency (f) for a source emitting light waves of frequency f_0 and moving with relative radial speed v (speed parameter $\beta = v/c$) is given by:

$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}$ (source and detector separating)

$f = f_0\sqrt{\frac{1+\beta}{1-\beta}}$ (source and detector moving toward)

a. How fast and in what direction must galaxy A be moving if an absorption line found at wavelength 550nm (λ_0) (green) for a stationary galaxy is shifted to 450 nm (blue) for galaxy A?

$\lambda_0 = 550$ nm, $\lambda = 450$ nm, we have a blue shift, $\boxed{\text{moving toward}}$

$$f = f_0\sqrt{\frac{1+\beta}{1-\beta}}$$

$$\frac{c}{\lambda} = \frac{c}{\lambda_0}\sqrt{\frac{1+\beta}{1-\beta}} \rightarrow \frac{\lambda_0}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow \frac{1+\beta}{1-\beta} = \left(\frac{\lambda_0}{\lambda}\right)^2 = \left(\frac{550}{450}\right)^2 = 1.4938$$

Solving for β: $1+\beta = 1.4938 - 1.4938\beta$

$\beta = 0.198$, $\boxed{v = 5.94 \times 10^7 \text{ m/s}}$

b. How fast and in what direction is galaxy B moving if it shows the same line shifted to 700 nm (red)?

$\lambda = 700$ nm, $\lambda_0 = 550$ nm, we have a red-shift.

$\boxed{\text{Moving away}}$

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}}$$

$$\frac{c}{\lambda} = \frac{c}{\lambda_0}\sqrt{\frac{1-\beta}{1+\beta}} \rightarrow \frac{\lambda_0}{\lambda} = \sqrt{\frac{1-\beta}{1+\beta}}$$

$$\left(\frac{550}{700}\right)^2 = \frac{1-\beta}{1+\beta} \rightarrow \beta = 0.237$$

$$\boxed{v = 7.097 \times 10^7 \text{ m/s}}$$