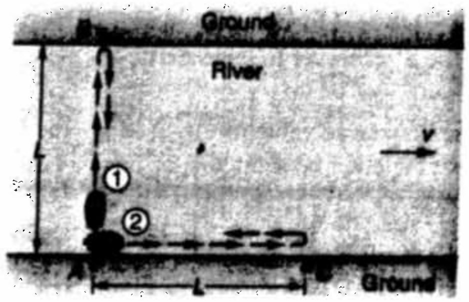
EXAMPLE 1-1 **A Boat Race** Two equally matched rowers race each other over courses as shown in Figure 1-6a. Each oarsman rows at speed c in still water; the current in the river moves at speed v. Boat 1 goes from A to B, a distance L, and back. Boat 2 goes from A to C, also a distance L, and back. A, B, and C are marks on the riverbank. Which boat wins the race, or is it a tie? (Assume $c > v$.)

(a)



(b)

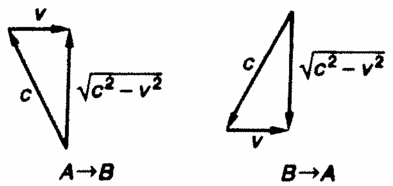


FIGURE 1-6 (a) The rowers both row at speed c in still water. The current in the river moves at speed v. Rower 1 goes from A to B and back to A, while rower 2 goes from A to C and back to A. (b) Rower 1 must point the bow upstream so that the sum of the velocity vectors $\mathbf{c} + \mathbf{v}$ results in the boat moving from A directly to B. His speed relative to the banks (i.e., points A and B) is then $(c^2 - v^2)^{1/2}$. The same is true on the return trip.

SOLUTION

The winner is, of course, the boat that makes the round trip in the shortest time, so to discover which boat wins, we compute the time for each. Using the classical velocity transformation (Equations 1-3), the speed of 1 relative to the ground is $(c^2 - v^2)^{1/2}$, as shown in Figure 1-6b; thus the round trip time t_1 for boat 1 is

$$t_1 = t_{A\to B} + t_{B\to A} = \frac{L}{\sqrt{c^2 - v^2}} + \frac{L}{\sqrt{c^2 - v^2}} = \frac{2L}{\sqrt{c^2 - v^2}}$$

$$= \frac{2L}{c\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2L}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx \frac{2L}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots\right) \qquad 1\text{-}4$$

where we have used the binomial expansion (see Appendix B2). Boat 2 moves downstream at speed $c + v$ relative to the ground and returns at $c - v$, also relative to the ground. The round trip time t_2 is thus

$$t_2 = \frac{L}{c + v} + \frac{L}{c - v} = \frac{2Lc}{c^2 - v^2}$$

$$= \frac{2L}{c}\frac{1}{1 - \frac{v^2}{c^2}} \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2} + \cdots\right) \qquad 1\text{-}5$$

/hich, you may note, is the same result obtained in our discussion of the speed of ght experiment in the Classical Concept Review.

The difference Δt between the round-trip times of the boats is then

$$\Delta t = t_2 - t_1 \approx \frac{2L}{c}\left(1 + \frac{v^2}{c^2}\right) - \frac{2L}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right) \approx \frac{Lv^2}{c^3} \qquad 1\text{-}6$$

he quantity Lv^2/c^3 is always positive; therefore, $t_2 > t_1$ and rower 1 has the faster /erage speed and wins the race.