PHYS 211 F 2013 Test #3 Name: KEY

1. An automobile has a total mass of 1300 kg, which includes 100 kg for the 4 wheels, each of which can be approximated as a uniform disk of diameter 66 cm. It starts from rest and the wheels turns through 22 revolutions in 12 seconds. Find the following at the end of 12 seconds. (1 revolution = 2π rad)

a. Angular speed of a wheel.
b. Linear speed of the automobile.
c. Translational kinetic energy of a wheel.
d. Rotational kinetic energy of a wheel.
e. Total kinetic energy of a wheel.
f. Total kinetic energy of the automobile.

a. $\omega_0 = 0$, $\omega = ?$, $\Delta\theta = 22$ rev. $= 22 \times 2\pi$ rad $= 44\pi$ rad, $t = 12$ Sec.

$$\Delta\theta = \frac{1}{2}(\omega + \omega_0)t$$

$$22 \times 2\pi = \frac{1}{2}(\omega + 0) \times 12 \longrightarrow \omega = \frac{22 \times 2\pi}{6} = 23 \text{ rad/s}$$

b. $v = r\omega = 0.33 \times 23 = 7.6$ m/s

c. $TKE = \frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 7.6^2 = 723\text{ J}$

d. $RKE = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = \frac{1}{4}mr^2\omega^2 = \frac{1}{4} \times 25 \times 0.33^2 \times 23^2$
$= 360\text{ J}$

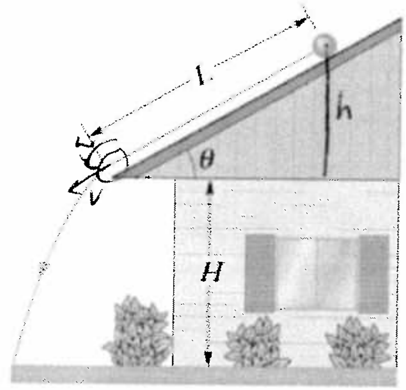
e. Total KE of a wheel $= 1083\text{ J}$

f. $\frac{1}{2}mv^2 + 4 \cdot \frac{1}{2}I\omega^2$

$\frac{1}{2} \times 1300 \times 7.6^2 + 4 \times 360 = 38984\text{ J}$

2. In the figure, a solid sphere starts from rest and rolls without slipping a distance $L = 6.0$ m down a roof that is inclined at the angle $\theta = 30°$. Use conservation of energy and find the linear speed of sphere as it leaves the roof?

$h = L\sin\theta = 6\sin 30° = 3\,m$



$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$mgh = \frac{1}{2}\left(\frac{2}{5}m\cancel{R^2}\right)\frac{v^2}{\cancel{R^2}} + \frac{1}{2}mv^2$

$gh = \frac{1}{5}v^2 + \frac{1}{2}v^2$

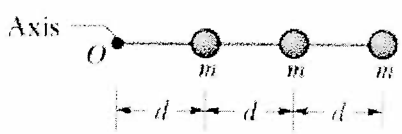
$gh = \frac{7}{10}v^2$

$v^2 = \frac{10gh}{7}$

$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10 \times 9.8 \times 3}{7}} = \sqrt{42}$

$\boxed{v = 6.48\ m/s}$

3. Figure below shows three particles (each of mass, m = 10 g) that have been glued to three rods. Each rod has a length, d = 2 cm and mass 20 g. The assembly can rotate around a perpendicular axis through point O at the left end. Calculate the rotational inertia of the assembly about the axis shown.



$I_{Tot} = I_{rod} + I_{masses}$

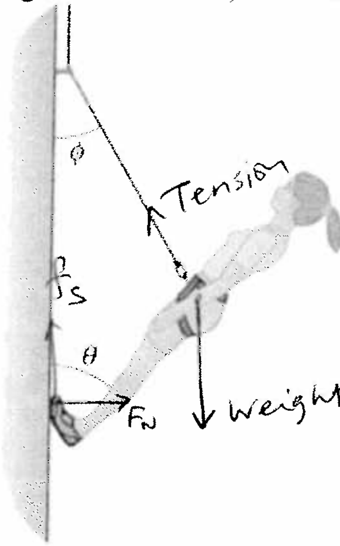
$= \frac{1}{3}(60)6^2 + 10 \times 2^2 + 10 \times 4^2 + 10 \times 6^2$

$= 720 + 40 + 160 + 360$

$I_{Tot} = 1280\ g\cdot cm^2 = 1.28 \times 10^{-4}\ kg\cdot m^2$

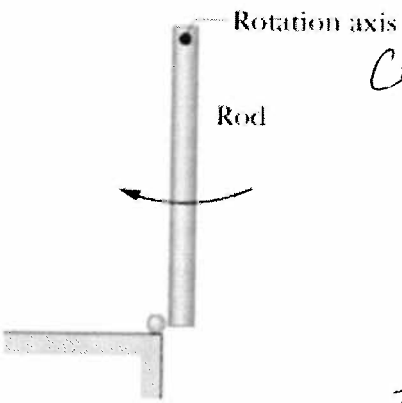
4. In the figure below, a climber is held by a belay rope connected to her climbing harness and belay device; the force of the rope on her has a line of action through her center of mass. Identify (naming and showing) the forces acting on the climber, in the figure.

f_s = static frictional force
F_N = normal force



ℓ m

5. The uniform rod (length 0.60 m, mass 1.2 kg) shown below, rotates in the plane of the figure about an axis through one end. As the rod swings through its lowest position, it collides with a 0.30 kg putty wad that sticks to the end of the rod. If the rod's angular speed just before collision is 2.4 rad/s, what is the angular speed of the rod–putty system immediately after collision?



Conservation of angular momentum.

$$L_i = L_f$$

$$I_{rod} \cdot w_i = (I_{rod} + I_{putty})\, w_f$$

$$\frac{1}{3} m\ell^2 \cdot w_i = \left(\frac{1}{3} m\ell^2 + m\ell^2\right) w_f$$

$$\frac{1}{3} \times 1.2 \times 0.6^2 \times 2.4 = \left(\frac{1}{3} \times 1.2 \times 0.6^2 + 0.3 \times 0.6^2\right) w_f$$

$$0.3456 = 0.252\, w_f$$

$$\boxed{w_f = 1.37 \text{ rad/s}}$$

6. At time t, $r = 2t^2 i - 3t^3 j + k$ gives the position of a 2.0 kg particle relative to the origin of an xy coordinate system ($\vec{r}$ is in meters and t is in seconds).

I. Find an expression as a function of time for a) the velocity b) the linear momentum c) the acceleration d) the force, of the particle relative to the origin.

II. About the origin, at t = 1s, determine d) the torque and e) the angular momentum of the particle in unit–vector notation.

III. At t = 1s, what is the angle between the position and linear momentum vectors.

I. $\vec{r} = 2t^2\hat{i} - 3t^3\hat{j} + \hat{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = 4t\hat{i} - 9t^2\hat{j}$

$\vec{p} = m\vec{v} = 2(4t\hat{i} - 9t^2\hat{j}) = 8t\hat{i} - 18t^2\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = 4\hat{i} - 18t\hat{j}$

$\vec{F} = m\vec{a} = 2(4\hat{i} - 18t\hat{j}) = 8\hat{i} - 36t\hat{j}$

II. $\vec{\tau}_{\text{at 1sec}} = \vec{r} \times \vec{F} = (2\hat{i} - 3\hat{j} + \hat{k}) \times (8\hat{i} - 36\hat{j})$

$= -72\hat{k} + 24\hat{k} + 8\hat{j} + 36\hat{i}$

$\vec{\tau} = (36\hat{i} + 8\hat{j} - 48\hat{k})$ N.m

$\vec{l}_{\text{at 1sec}} = \vec{r} \times \vec{p} = (2\hat{i} - 3\hat{j} + \hat{k}) \times (8\hat{i} - 18\hat{j})$

$= -36\hat{k} + 24\hat{k} + 8\hat{j} + 18\hat{i}$

$\vec{l} = (18\hat{i} + 8\hat{j} - 12\hat{k})$ $\text{Kg}.\frac{m^2}{s}$

III. $l = \vec{r} \times \vec{p} = rp\sin\theta$

$\sin\theta = \frac{|\vec{r} \times \vec{p}|}{rp} = \frac{|\vec{l}|}{rp} = \frac{\sqrt{18^2 + 8^2 + (-12)^2}}{\sqrt{2^2 + (-3)^2 + 1^2} \times \sqrt{8^2 + (-18)^2}}$

$= 0.3129$

$\theta = \sin^{-1}(0.3129) = \underline{\underline{18.2^\circ}}$