PHYSICS 201 T3 Equations Sheet F 2018 12:30	Translational Motion	Rotational Motion
	LINEAR	ANGULAR
Time	t	T
Displacement	x; $(x = r\theta)$	θ
Velocity	$v = \Delta x/\Delta t$; $(v = r\omega)$	$\omega = \Delta\theta/\Delta t$
Acceleration	$a = \Delta v/\Delta t$; $(a = r\alpha)$	$\alpha = \Delta\omega/\Delta t$ $(a_c = r\omega^2 = \frac{v^2}{r})$
Kinematic Equations	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
	$x = ½(v + v_0)t$	$\theta = ½(\omega + \omega_0)t$
	$x = v_0 t + ½ at^2$	$\theta = \omega_0 t + ½ \alpha t^2$
	$v^2 = v_0^2 + 2ax$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
Inertia	m = mass	I = Rotational inertia; $I = \sum m_i r_i^2$
To create	force = F	torque = τ = LA· F
Newton's second law of motion	$\Sigma \mathbf{F} = m\mathbf{a}$	$\Sigma \boldsymbol{\tau} = I\boldsymbol{\alpha}$
	$\Sigma \mathbf{F} = \Delta \mathbf{p}/\Delta t$	$\Sigma \boldsymbol{\tau} = \Delta \mathbf{L}/\Delta t$
Work	$F \cdot x$	$\tau \cdot \theta$
Kinetic Energy	Translational Kinetic Energy = TKE = ½ mv^2	Rotational Kinetic Energy = RKE = ½ $I\omega^2$
Momentum	$\mathbf{p} = m \cdot \mathbf{V}$	$\mathbf{L} = I \cdot \boldsymbol{\omega}$
Conservation of momentum	$\Sigma m_i v_i = \Sigma m_f v_f$	$\Sigma I_i \omega_i = \Sigma I_f \omega_f$

Acceleration due to gravity = g = 9.8 m/s^2. 1 Revolution = 2π rad.

Area of a circle of radius r, $A_{circle} = \pi r^2$ .Area of a rectangle of length l, and width w, A_{rec}=l x w; Area of a triangle, $A_{triangle}$= 0.5 x base x height.

Volume of a cylinder of radius r and height h; V= $\pi r^2 h$; Volume of a sphere = (4/3) πr^3.
Frictional force = $F_{fr} = \mu_k F_N$ Buoyant force: $F_b = \rho_f v_f g$ GPE = mgh
Hooke's law: $\vec{F} = -k\vec{x}$ Elastic PE = EPE = $\frac{1}{2}kx^2$ Period = 1/Frequency

Period of a simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ Period of oscillating mass on spring: $T = 2\pi\sqrt{\frac{m}{k}}$

The moment of inertia, I for a cylinder (or disk) of mass m and radius r is: $I = \frac{1}{2}mr^2$

PHYS 201 Fall 2018 Test #3 Name: KEY

I. For the following multiple choice questions, write your answer in the line next to the question number.

a 1. What is the angular speed in degree/second of the second hand of an analog watch?

a. 6 b. 12 c. 15 d. 30 e. 36

$\frac{360°}{60 s} = 6 \frac{deg}{s}$

b 2. What is the angular speed in rad/s of the second hand of an analog watch?

a. 1.75×10^{-3} b. 0.105 c. 8.33×10^{-3} d. 8.73×10^{-3} e. 1.45×10^{-4}

$\frac{2\pi \text{ rad}}{60 s}$

b 3. The radius of each wheel on a bicycle is 0.400 m. The bicycle travels a distance of 3.0 km. How many revolutions does each wheel make (wheels do not slip)?

a. 7.5 b. 1200 c. 2400 d. 6000 e. 7500

$x = r\theta$, $3000 = 0.4\theta$, $\theta = 7500$ rad, $\theta = \frac{7500}{2\pi}$ rev $= 1193$ rev ≈ 1200

a 4. A ball of radius 0.200 m is given an initial angular velocity of 15 rad/s. The ball rolls along a straight line for 5 seconds until it comes to rest. How far the ball travels during this time?

a. 7.5 m b. 37.5 m c. 3 m d. 75 m e. 15 m f. 1.0 m

$\omega_0 = 15$ rad/s, $\omega = 0$, $t = 5s$, $\theta = \frac{1}{2}(\omega + \omega_0)t = \frac{1}{2}(15) \times 5 = 37.5$ rad

$x = r\theta = 0.2 \times 37.5$, $x = 7.5$ m

d 5. A figure skater is spinning with an angular velocity of 18 rad/s. She then comes to a stop over a brief period of time. During this time, her angular displacement is 5.1 rad. Determine her average angular acceleration, in SI units.

a. 3.5 rad/s^2 b. 32 rad/s^2 c. -3.5 rad/s^2 d. -32 rad/s^2

$\omega_0 = 18$ rad/s, $\omega = 0$, $\theta = 5.1$ rad

$\omega^2 = \omega_0^2 + 2\alpha\theta$, $0 = 18^2 + 2\alpha \times 5.1$

$\alpha = -18^2/(2 \times 5.1) = -31.76 = -32$

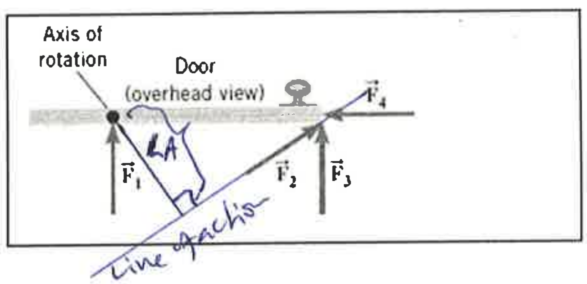
6-7) The drawing illustrates an overhead view of a door and its axis of rotation. The axis is perpendicular to the page. There are four forces acting on the door, and they have the same magnitude.

6. Show the lever-arm for the force F_2 in the diagram.

d 7. Which pair of forces provide non-zero torque, about the axis of rotation shown?

a. $\mathbf{F_1}$ and $\mathbf{F_2}$ b. $\mathbf{F_3}$ and $\mathbf{F_4}$ c. $\mathbf{F_1}$ and $\mathbf{F_4}$

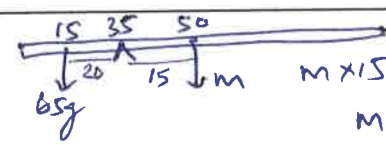
d. $\mathbf{F_3}$ and $\mathbf{F_2}$ e. $\mathbf{F_1}$ and $\mathbf{F_3}$



8-9) A uniform meter stick is supported at the 35 cm mark. Balance is obtained when a 65 gram mass is suspended at the 15 cm mark.

8. Draw a free-body diagram for the meter stick.

8.



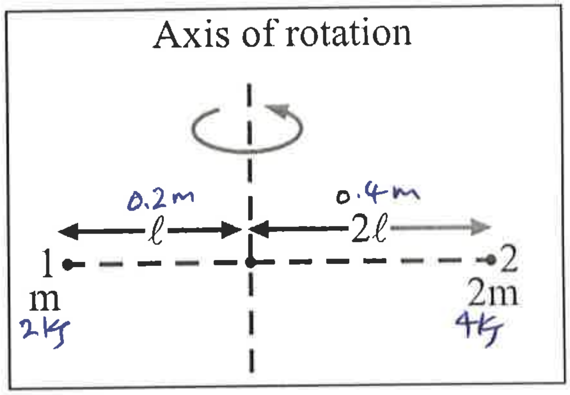
$m \times 15 = 65 \times 20$

$m = \frac{65 \times 20}{15}$

$m = 86.7$ g

d 9. What is the mass of the meter stick?

a. 29 g b. 46 g c. 75 g d. 87 g e. 93 g



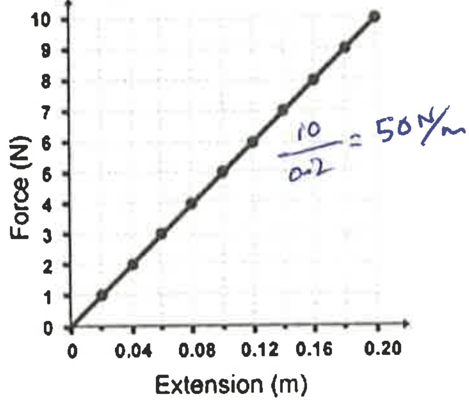
d 10. Two 'point' masses, 1 has a mass of 2 kg and 2 has a mass of 4 kg, rotated as shown. The length l = 0.2 m and 2l = 0.4 m. Calculate the moment of inertia about the axis of rotation shown?

a. 0 b. $0.08\ kg.m^2$ c. $0.64\ kg.m^2$
d. $0.72\ kg.m^2$ e. $2.72\ kg.m^2$

$I = 2\times(0.2)^2 + 4\times(0.4)^2$
$I = 0.72\ kg.m^2$

f 11. The stretching force VS. stretch curve for a Hooke's law experiment is shown below. What is the spring constant of the spring?

a. 5 N/m
b. 10 N/m
c. 15 N/m
d. 20 N/m
e. 25 N/m
f. 50 N/m



c 12. What is the period, in millisecond, of the sound wave of frequency 440 Hz?
a. 0.0023 b. 0.023 c. 2.3 d. 4.4 e. 44

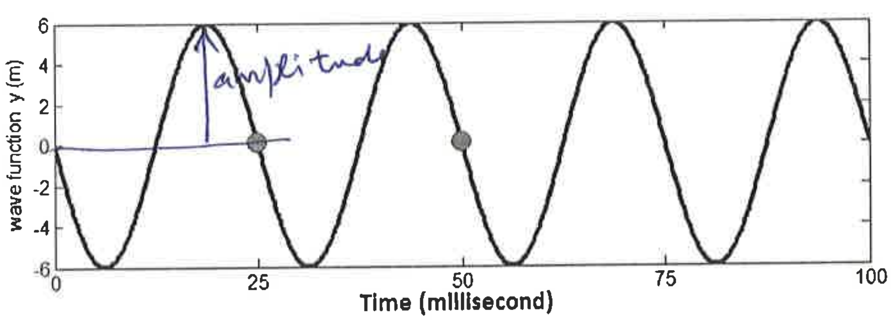
$T = \frac{1}{f} = 0.002275$
$= 2.27\ ms \approx 2.3\ ms$

d 13. The period of a simple pendulum of length 0.993 m is found to be 1.95 s at a location. Calculate the acceleration due to gravity at this location to 3 significant figures.
a. $9.80\ m/s^2$ b. $9.83\ m/s^2$ c. $9.90\ m/s^2$ d. $10.3\ m/s^2$ e. $10.6\ m/s^2$

$T = 2\pi\sqrt{\frac{L}{g}}$
$g = \frac{4\pi^2 L}{T^2}$

C 14. The spring loaded gun in a ballistic pendulum is compressed by 5.2 cm and released. The 54 gram steel ball leaves the gun with a velocity of 6.3 m/s. Calculate the spring constant of the spring.
a. 7.93 N/m b. 79.3 N/m c. 793 N/m d. 7930 N/m

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$
$k = \frac{mv^2}{x^2}$
$k = \frac{0.054 \times 6.3^2}{0.052^2}$



a 15. What is the amplitude, in m, for the wave shown above?
d 16. What is the frequency in Hz for the wave shown above?
Answers for 15 & 16: a. 6 b. 12 c. 25 d. 40 e. 80

$T = 25\ ms = 0.025\ s$
$f = \frac{1}{T} = \frac{1}{0.025} = 40\ Hz$

II. Automobile Chevy volt, mass = 1750 kg (including its 4 wheels), moving with a velocity of 35 m/s.

a. Calculate the translational kinetic energy of the car in SI units.

b. If each of the rolling wheels, assumed to be a uniform disk, has a mass of 15 kg and radius 0.30 m, calculate the rotational kinetic energy for one wheel.

c. Calculate the total (rotational & translational) kinetic energy of the car.

a. $TKE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1750 \times 35^2 = 1071875\,J \cdot 1.07\times10^6\,J$

b. $RKE = \frac{1}{2}I\omega^2$, $\omega = \frac{v}{r} = \frac{35}{0.3} = 116.6$ rad/s, $I = 0.675\,kg.m^2$

$= \frac{1}{2} \times \frac{1}{2} \times m r^2 \times \omega^2$

$RKE = \frac{1}{2} \times \frac{1}{2} \times 15 \times 0.3^2 \times 116.6^2$

$RKE = 4593.75\,J$ ← one wheel

c. $TKE + 4 \times RKE$

$1071875 + 4 \times 4593.75 = 1090250\,J$

$Tot.\,KE = 1.09 \times 10^6\,J$

III. A playground merry-go-round (a disk) has a mass of 120 kg and a radius of 2.0 m and it is rotating with an angular velocity of 3.5 rad/s. What is its angular velocity after a 15 kg child gets onto it by grabbing its outer edge? The child is initially at rest. Express your answer in SI units.

$I_1 \omega_1 = I_2 \omega_2$

$I_1 = \frac{1}{2}mr^2 = \frac{1}{2} \times 120 \times 2^2 = 240\,kg.m^2$

$I_2 = I_1 + mr^2 = 240 + 15 \times 2^2 = 300\,kg.m^2$

$240 \times 3.5 = 300 \times \omega_2$

$\omega_2 = \frac{240 \times 3.5}{300} = 2.8$ rad/s

$\omega_2 = 2.8$ rad/s

IV. Three forces are applied to a solid cylinder of mass 15 kg (see the drawing). The magnitudes of the forces are $F_1 = 15$ N, $F_2 = 24$ N, and $F_3 = 16$ N. The radial distances are $R_2 = 0.26$ m and $R_3 = 0.12$ m. The forces $\mathbf{F}_2$ and $\mathbf{F}_3$ are perpendicular to the radial lines labeled R_2 and R_3. Find the magnitude of the angular acceleration of the cylinder about the axis of rotation.

$\Sigma\tau = F_2 R_2 - F_3 R_3 + F_1 \times 0$

$\Sigma\tau = 24 \times 0.26 - 16 \times 0.12$

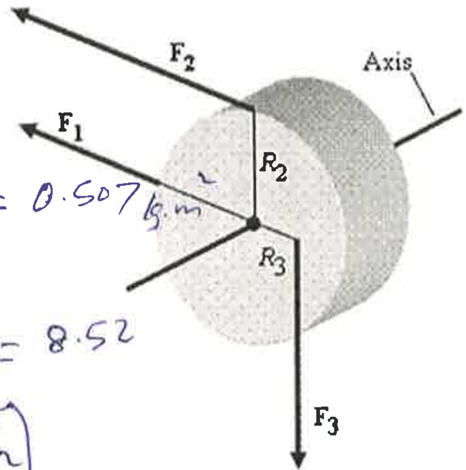
$\Sigma\tau = 4.32$ N·m

$I = \frac{1}{2} m r^2 = \frac{1}{2} \times 15 \times (0.26)^2 = 0.507$ kg·m²

$\Sigma\tau = I\alpha$

$\alpha = \frac{\Sigma\tau}{I} = \frac{4.32}{.507} = 8.52$

$\alpha = 8.52$ rad/s²



V. In an isometric exercise a person places a hand on a scale and pushes vertically downward, keeping the forearm horizontal. This is possible because the triceps muscle applies an upward force, **M** perpendicular to the arm, as the drawing indicates. The forearm weighs 19.5 N and has a center of gravity as indicated. The scale registers 108 N. Draw a free-body diagram for the forearm and determine the magnitude of **M**. Express your answer in SI units with 3 significant figures.

$\Sigma\tau = 0$, about the Elbow joint

$M \times 0.025 + 19.5 \times 0.15 + 108 \times 0.3 = 0$
(CW) (CW) (CCW)

$M \times 0.025 + 19.5 \times 0.15 = 108 \times 0.3$

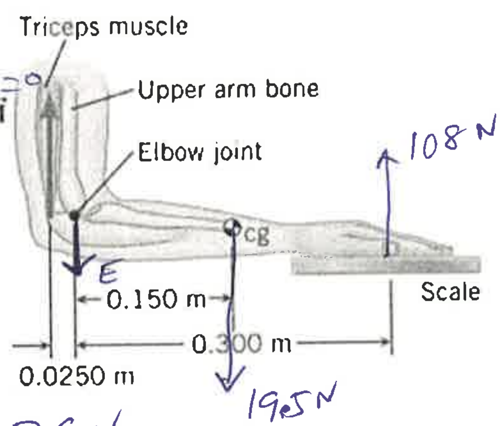
$0.025M + 2.925 = 32.4$

$0.025M = 32.4 - 2.925$

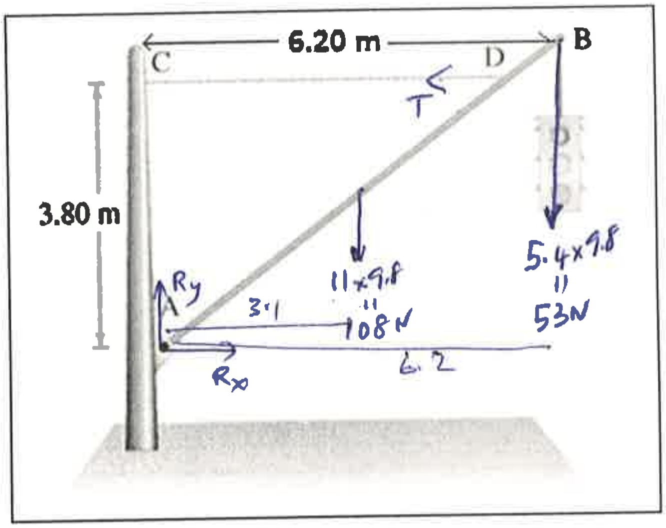
$= 29.475$

$M = \frac{29.475}{.025} = 1179$ N

$\vec{M} = 1180$ N



VI. A traffic light of mass 5.4 kg is supported by a uniform pole (AB) of mass 11 kg, hinged at A, and a horizontal cable, CD. Draw a free-body diagram for the pole, identifying and showing all the forces acting on it, and determine the tension in the cable and the horizontal and vertical forces exerted by the hinge on the pole.



$\sum F_x = 0$

$R_x - T = 0 \rightarrow R_x = T$

$\sum F_y = 0$

$R_y - 108 - 53 = 0$

$R_y = 108 + 53 = 161\,N$

$\sum \tau = 0$ about the hinge

$108 \times 3.1 + 53 \times 6.2 + T \times 3.8 = 0$
(cw) (cw) (ccw)

$108 \times 3.1 + 53 \times 6.2 = 3.8\,T$

$663.4 = 3.8\,T$

$T = \frac{663.4}{3.8} = 175\,N$

$R_x = T = 175\,N$

$R_x = 175\,N$
$R_y = 161\,N$
$T = 175\,N$