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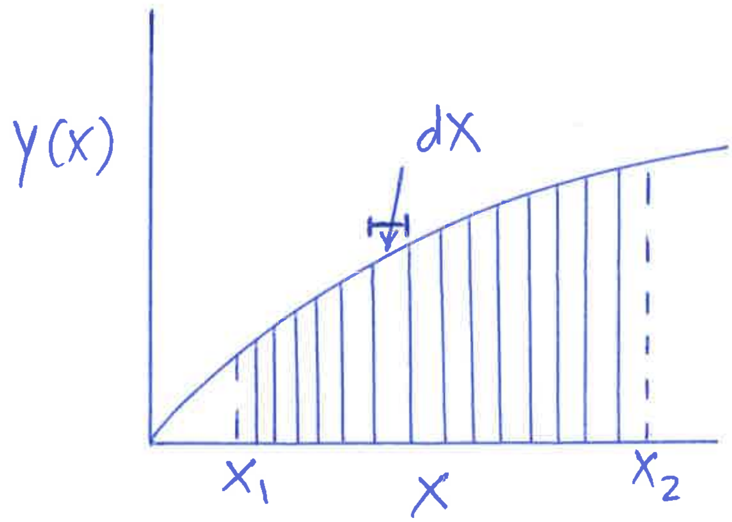
Evaluating dx, Δx & ∂x

— The term dx usually signifies a very small, infinitesimal change in x. Δx is the sum of all horizontal elements dx within a defined range:



$$\Delta x = x_2 - x_1$$

— The total area of the region under the curve in the interval of Δx is the integral of $y(x)$:

$$\text{area} = \int_{x_1}^{x_2} y(x)\,dx = \text{sum of all } \underline{\underline{\text{area}}} \text{ element } y(x)\cdot dx$$

— The rate at which the dependent variable, $y(x)$ changes as x changes is the <u>derivative</u>. If y depends only on x, we can represent this as:

$$y'(x) = \frac{dy}{dx}$$

② High order derivatives

— Derivatives can be taken more than once to express the change in a rate of change.

· Ex. velocity is the rate of change in position with time:

$$v(t) = \frac{dx(t)}{dt}$$

Acceleration is the rate of change in velocity:

$$A(t) = \frac{dv(t)}{dt} = \frac{d}{d}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

③ Derivative review

- product rule : $\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$
- quotient rule : $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{(g(x))^2}$
- chain rule: $\frac{d}{dx}\left(f(g(x))\right) = \left(\frac{df}{du}\right)\left(\frac{du}{dx}\right)$

~~*~~ requires $\underline{u}$ substitution

- If y depends on more than one variable (ex. $y(x,z)$), then the derivative of $y(x,z)$ with respect to x is a partial derivative at constant z: $\left(\frac{\partial y}{\partial x}\right)_z = \ldots$

- For example, in the ideal gas law:

  $PV = nRT$

  P can be considered a function of V & T:

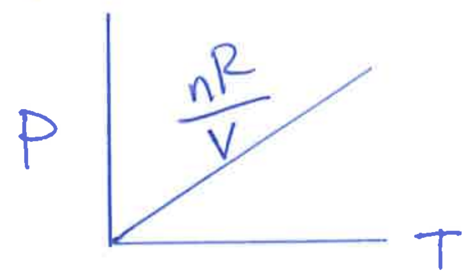
  $$P(V,T) = \frac{nRT}{V}$$

  - If we wanted to know how P changes with T at constant V, we would express the partial derivative of P:

  $$\left(\frac{\partial P(V,T)}{\partial T}\right)_V = \frac{\partial}{\partial T}\left(\frac{nRT}{V}\right)_V$$

  * Since V is constant...

  $$\frac{\partial}{\partial T}\left(\frac{nRT}{V}\right)_V = \frac{nR}{V}\frac{\partial}{\partial T}(T) = \frac{nR}{V}$$



ex. $f(x) = \frac{1}{\sqrt{1-x^2}}$ Find $\frac{df}{dx}$

This is a function of a function!

$g(x) = 1-x^2$ $\quad u = 1-x^2$

$f(g(x)) = \frac{1}{\sqrt{g(x)}}$ $\quad \frac{du}{dx} = -2x$

$= \frac{1}{\sqrt{u}}$ $\quad \frac{df}{du} = \frac{d}{du} u^{-\frac{1}{2}} = -\frac{1}{2} u^{-\frac{3}{2}}$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = \left(-\frac{1}{2} u^{-\frac{3}{2}}\right)\left(-2x\right)$$

$$\frac{df}{dx} = u^{-\frac{3}{2}} \cdot x$$

$$\boxed{\frac{df}{dx} = \frac{x}{(1-x^2)^{\frac{3}{2}}}}$$

Ex. $\frac{d}{dx} \sin(x^2)$

Ans: $2x\cos x^2$

chain rule

$\frac{d}{dx} \sin^2 x$

Ans: $2\cos x \sin x$

product

* $\frac{d}{dx}\left(\sqrt{\frac{2+x}{2-x}}\right)$

Ans: $\frac{2}{(2-x)^{\frac{3}{2}}\sqrt{2+x}}$

chain & quotient

④ Integration

- Refer to an integral table for common integrals * $\boxed{\int f'(x)\,dx = f(x)}$

— For multiplied functions, recall product rule:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)g'(x) + g(x)f'(x)$$

If we integrate both sides:

$$\int \frac{d}{dx}\left(f(x)\cdot g(x)\right)dx = \int\left[f(x)g'(x)+g(x)f'(x)\right]dx$$

$$f(x)g(x) = \int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx$$

$$\boxed{\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx}$$

integration by parts!!

Ex. $\int x\sin x\,dx$ let $f(x) = x$ $g'(x) = \sin x\,dx$

$f'(x) = 1$ $g(x) = -\cos x$

$$\int x\sin x\,dx = -x\cos x - \int -\cos x\,(1)\,dx$$

$$= -x\cos x + \sin x + C$$

ex. $\int t^2 e^t \, dt$

Always set function "f" as the term having a numerical power. this will make the problem solvable because each derivative of f will lower the order

$f(t) = t^2$

$g'(t) = e^t \, dt$

$f'(t) = 2t$

$g(t) = e^t$

$$\int t^2 e^t \, dt = t^2 e^t - \int 2t e^t \, dt$$

$$= t^2 e^t - 2 \int t e^t \, dt$$

$f(t) = t \quad f'(t) = 1$

$g'(t) = e^t \, dt \quad g(t) = e^t$

$$= t^2 e^t - 2 \left[t e^t - \int e^t \, dt \right]$$

$$= t^2 e^t - 2 \left[t e^t - (e^t + C) \right]$$