Modern Physics **EXAM 2**

Name KEY

Problem1: 20 points

A free electron has a wave function

$\psi(x,)=A \sin(2.0\times10^{10}\, x)$ where x is given in meters. Determine the electron's

(a) wavelength,

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{2.0\times10^{10}\ m^{-1}} = 3.142\times10^{-10}\ m$$ 5

(b) momentum,

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ J.s}{3.142\times10^{10}\ m} = 2.11\times10^{-24}\ kg\,m/s$$ 5

(c) speed,

$$v = \frac{p}{m} = \frac{2.11\times10^{-24}\ kg\,m/s}{9.11\times10^{-31}\ kg} = 2.3\times10^{6}\ m/s$$ 5

and (d) kinetic energy in eV.

$$KE = \frac{p^2}{2m} = \frac{(2.11\times10^{-24}\ kg\,m/s)^2}{2(9.11\times10^{-31}\ kg)} \times \frac{1}{1.6\times10^{-19}\ J/eV}$$

5

$$= 15\, eV$$

Modern Physics **EXAM 3**
Spring 2019

Name________________________________

Problem 2: 20 points

Show that for a free particle of mass m moving in one dimension, the function

$\psi(x) = A\cos kx + B\sin kx$ is a solution to the time-independent Schrödinger equation for any values of the constants A and B.

Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi(x) \quad \text{for a free particle} \quad 5$$

$$\frac{\partial\psi(x)}{\partial x} = -kA\sin kx + kB\cos kx \quad 5$$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2 A\cos kx - k^2 B\sin kx$$

$$= -k^2\left[A\cos kx + B\sin kx\right] = -k^2\psi(x) \quad 5$$

we also know that:

$$p = \hbar k = \sqrt{2mE} \Rightarrow \hbar^2 k^2 = 2mE \quad 5$$

So

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = -\frac{\hbar^2}{2m}\left[-k^2\left(A\cos kx + B\sin kx\right)\right]$$

$$= \frac{-\hbar^2 k^2}{2m}\left[A\cos kx + B\sin kx\right] = \frac{\cancel{2m}E}{\cancel{2m}}\psi(x) = E\psi(x)$$

Modern Physics **EXAM 3**
Spring 2019

Name________________________________

Problem 3: What is the longest wavelength light capable of ionizing a hydrogen atom in the ground state? (20 points)

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$ 5

$$E_{in} = -\frac{13.6eV}{1} = -13.6eV$$ 5

$$\lambda = \frac{hc}{E_{in}} = \frac{(6.63 \times 10^{-3} J.s)(3.0 \times 10^{8} m/s)}{(13.6eV)(1.6 \times 10^{-19} J/eV)} = 9.14 \times 10^{-8} m$$

10

$$\boxed{\lambda = 91.4nm}$$

Modern Physics **EXAM 3**
Spring 2019

Name____________________________________

Problem 4: A free neutron ($m=1.67\times10^{-27}$kg) has a mean life of 900s. What is the uncertainty in its mass (in kg)? (20 points)

$\Delta E \Delta t \sim \hbar$ 5

$\Delta E = (\Delta m) c^2$ 5

$(\Delta m) c^2 \Delta t \sim \hbar$ 5

$\Rightarrow \Delta m \sim \frac{\hbar}{c^2 \Delta t} = \frac{(1.055\times10^{-34}\,J\cdot s)}{(3.0\times10^{8}\,m/s)^2 (900\,s)} = 1.3\times10^{-54}\,kg$

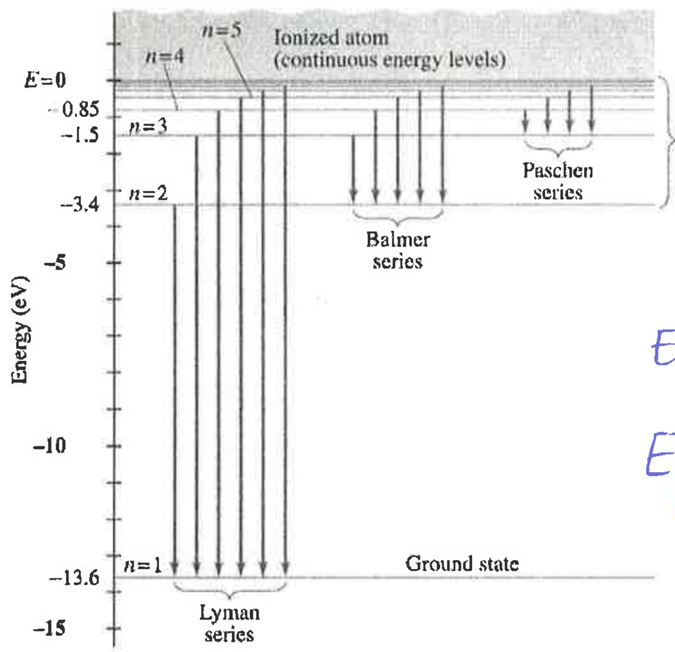
5

Modern Physics **EXAM 3**
Spring 2019

Name________________________________

Problem 5: (20 points)

(*a*) Determine the wavelength of the second Balmer line ($n=4$ to $n=2$ transition) using Fig. below.



$E_n - E_m = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{E_n - E_m}$

2nd Balmer = −0.85eV

(8)

$E_4 = -\frac{13.6eV}{4^2} = 0.85eV$

$E_2 = \frac{-13.6eV}{2^2} = -3.4eV$

$\lambda = \frac{hc}{E_4 - E_2} = \frac{1240eVnm}{0.85-(-3.4eV)}$

$\lambda = 486.27 \times 10^{-9}m$

Determine likewise (*b*) the wavelength of the third Lyman line

3rd Lyman is − 0.85eV $n=1 \Rightarrow E_1 = -13.6eV$

$\lambda = \frac{1240eVnm}{(-0.85eV)-(-13.6eV)} = 97.3nm$

(6)

and (*c*) the wavelength of the first Balmer line.

first Balmer $n=3 \rightarrow n=2$

$E_3 = -\frac{13.6eV}{3^2} = -1.5eV$ $E_2 = \frac{-13.6eV}{2^2} = -3.4eV$

(6)

$\lambda = \frac{1240eVnm}{-1.5eV-(-3.4eV)} = 650nm$