Modern Physics **EXAM 1** **Fall 2022**

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Problem 1: (I) What is the momentum of a proton traveling at $v=0.75c$ ($m_p=1.67\times10^{-27}$ kg)

$$P = \gamma m v = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{(1.67\times10^{-27}kg)(0.75c)}{\sqrt{1-0.75^2\frac{\cancel{c^2}}{\cancel{c^2}}}}$$

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$$= \frac{(1.67\times10^{-27}kg)(0.75)(3\times10^8 m/s)}{\sqrt{1-0.75^2}} = 5.7\times10^{-19}\ kg\cdot m/s$$

Problem 2: A particle travels at $v=0.10c$. By what percentage will a calculation of its momentum be wrong if you use the classical formula?

$$\frac{P_{classical}}{P_{relativity}} = \frac{\cancel{m}v}{\gamma \cancel{m}v} = \frac{1}{\gamma} = \frac{1}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} = \sqrt{1-\frac{v^2}{c^2}}$$

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$$= \sqrt{1-0.10^2\frac{\cancel{c^2}}{c^2}} = \sqrt{1-0.10^2} = 0.995$$

The classical momentum is about:

$1-0.995 = 0.005 \Rightarrow 0.5\%$ in error

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Problem 3: A certain type of elementary particle travels at a speed of $2.70 \times 10^8 m/s$ .At this speed, the average lifetime is measured to be 4.76 ×10⁻⁶s. What is the particle's lifetime at rest?

$$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma} \Rightarrow \Delta t_0 = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Delta t_0 = 4.76 \times 10^{-6} s \sqrt{1 - \left(\frac{2.70 \times 10^8 m/s}{3 \times 10^8 m/s}\right)^2} = 2.07 \times 10^{-6} s$$

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$$\boxed{\Delta t_0 = 2.07 \times 10^{-6} s}$$

Problem 4: What is the speed of a pion if its average lifetime is measured to be $4.40 \times 10^8 s$. At rest, its average lifetime is $2.60 \times 10^8 s$

$$\Delta t_0 = 2.60 \times 10^8 s$$

$$\Delta t = 4.40 \times 10^8 s$$

$$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma} = \Delta t \sqrt{1 - \frac{v^2}{c^2}}$$

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$$\frac{\Delta t_0}{\Delta t} = \sqrt{1 - \frac{v^2}{c^2}} \Rightarrow \left(\frac{\Delta t_0}{\Delta t}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2 \Rightarrow$$

$$v^2 = c^2\left(1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2\right) \Rightarrow v = c\sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}$$

$$v = c\sqrt{1 - \left(\frac{2.60 \times 10^8 s}{4.40 \times 10^8 s}\right)^2} = 0.807c$$

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Problem 5: Suppose a news report stated that starship *Enterprise* had just returned from a 5-year voyage while traveling at 0.74*c*.

(*a*) If the report meant 5.0 years of *Earth time*, how much time elapsed on the ship?

$$\Delta t = \gamma \Delta t_0 \Rightarrow \Delta t_0 = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1-\frac{v^2}{c^2}}$$

$$= (5.0\,yr)\sqrt{1-\frac{(0.74)^2 \cancel{c^2}}{\cancel{c^2}}} = 3.4 \text{ years}$$

$$\boxed{\Delta t_0 = 3.4 \text{ years}}$$

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(*b*) If the report meant 5.0 years of *ship time*, how much time passed on Earth?

$$\Delta t_0 = \frac{\Delta t}{\gamma} \Rightarrow \Delta t = \gamma \Delta t_0 = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{5.0 \text{ years}}{\sqrt{1-(0.74)^2\frac{c^2}{c^2}}}$$

$$\boxed{\Delta t = 7.4 \text{ years}}$$

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