Exam 1 review-part 2

Double-Slit Interference

2. (I) Monochromatic light falling on two slits 0.018 mm apart produces the fifth-order bright fringe at a 9.8° angle. What is the wavelength of the light used?

For constructive interference, the path difference is a multiple of the wavelength, Apply this to the fifth order.

$$d\sin\theta = m\lambda \quad \rightarrow \quad \lambda = \frac{d\sin\theta}{m} = \frac{\left(1.8\times10^{-5}\,\text{m}\right)\sin 9.8^\circ}{5} = \boxed{6.1\times10^{-7}\,\text{m}}$$

3. (I) The third-order bright fringe of 610 nm light is observed at an angle of 28° when the light falls on two narrow slits. How far apart are the slits?

For constructive interference, the path difference is a multiple of the wavelength,
Apply this to the third order.

$$d\sin\theta = m\lambda \quad \rightarrow \quad d = \frac{m\lambda}{\sin\theta} = \frac{3\left(610\times10^{-9}\,\text{m}\right)}{\sin 28^\circ} = \boxed{3.9\times10^{-6}\,\text{m}}$$

4. (II) Monochromatic light falls on two very narrow slits 0.048 mm apart. Successive fringes on a screen 6.00 m away are 8.5 cm apart near the center of the pattern. Determine the wavelength and frequency of the light.

For constructive interference, the path difference is a multiple of the wavelength, The location on the screen is given by $x = l\tan\theta$,. For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$. Adjacent fringes will have $\Delta m = 1$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m\ell}{d}$$

$$x_1 = \frac{\lambda m_1\ell}{d} \;;\; x_2 = \frac{\lambda(m+1)\ell}{d} \;\rightarrow\; \Delta x = x_2 - x_1 = \frac{\lambda(m+1)\ell}{d} - \frac{\lambda m\ell}{d} = \frac{\lambda\ell}{d}$$

$$\lambda = \frac{d\Delta x}{\ell} = \frac{(4.8\times10^{-5}\,\text{m})(0.085\,\text{m})}{6.00\,\text{m}} = \boxed{6.8\times10^{-7}\,\text{m}} \;;\; f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\,\text{m/s}}{6.8\times10^{-7}\,\text{m}} = \boxed{4.4\times10^{14}\,\text{Hz}}$$

5. (II) If 720-nm and 660-nm light passes through two slits 0.68 mm apart, how far apart are the second-order fringes for these two wavelengths on a screen 1.0 m away?

For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = \ell\tan\theta,$). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$. Second order means $m = 2$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1 m\ell}{d} \;;\; x_2 = \frac{\lambda_2 m\ell}{d} \;\rightarrow$$

$$\Delta x = x_2 - x_1 = \frac{(\lambda_2 - \lambda_1)m\ell}{d} = \frac{\left[(720-660)\times10^{-9}\,\text{m}\right](2)(1.0\,\text{m})}{(6.8\times10^{-4}\,\text{m})} = 1.76\times10^{-4}\,\text{m} \approx \boxed{0.2\,\text{mm}}$$

This justifies using the small angle approximation, since $x = \ell$.

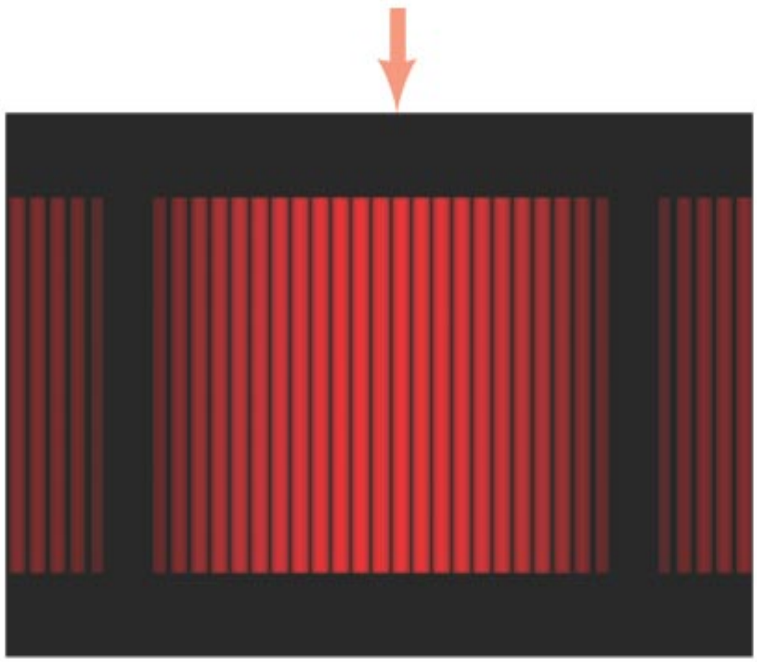
6. (II) A red laser from the physics lab is marked as producing 632.8-nm light. When light from this laser falls on two closely spaced slits, an interference pattern formed on a wall several meters away has bright fringes spaced 5.00 mm apart near the center of the pattern. When the laser is replaced by a small laser pointer, the fringes are 5.14 mm apart. What is the wavelength of light produced by the pointer?

The slit spacing and the distance from the slits to the screen is the same in both cases. The distance between bright fringes can be taken as the position of the first bright fringe (m = 1) relative to the central fringe. We indicate the lab laser with subscript 1, and the laser pointer with subscript 2. For constructive interference, the path difference is a multiple of the wavelength, The location on the screen is given by $x = \ell\tan\theta,$ as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/\ell$.

$$d\sin\theta = m\lambda \;\rightarrow\; d\frac{x}{\ell} = m\lambda \;\rightarrow\; x = \frac{\lambda m\ell}{d} \;;\; x_1 = \frac{\lambda_1\ell}{d} \;;\; x_2 = \frac{\lambda_2\ell}{d} \;\rightarrow$$

$$\lambda_2 = \frac{d}{\ell}x_2 = \frac{\lambda_1}{x_1}x_2 = (632.8\,\text{nm})\frac{5.14\,\text{mm}}{5.00\,\text{mm}} = 650.52\,\text{nm} \approx \boxed{651\,\text{nm}}$$

7. (II) Light of wavelength λ passes through a pair of slits separated by 0.17 mm, forming a double-slit interference pattern on a screen located a distance 35 cm away. Suppose that the image in Fig. Below is an actual-size reproduction of this interference pattern. Use a ruler to measure a pertinent distance on this image; then utilize this measured value to determine $\lambda(nm)$



Constructive interference

Using a ruler on Fig. below, the distance from the $m = 0$ fringe to the $m = 10$ fringe is found to be about 13.5 mm. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{l} = m\lambda \rightarrow \lambda = \frac{dx}{ml} = \frac{dx}{ml} = \frac{(1.7\times10^{-4}\,\text{m})(0.0135\,\text{m})}{(10)(0.35\,\text{m})} = \boxed{6.6\times10^{-7}\,\text{m}}$$

8. (II) Light of wavelength 680 nm falls on two slits and produces an interference pattern in which the third-order bright fringe is 38 mm from the central fringe on a screen 2.6 m away. What is the separation of the two slits?

8. For constructive interference, the path difference is a multiple of the wavelength. The location on the screen is given by $x = l \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{l} = m\lambda \rightarrow d = \frac{\lambda m l}{x} = \frac{(680\times10^{-9}\,\text{m})(3)(2.6\,\text{m})}{38\times10^{-3}\,\text{m}} = \boxed{1.4\times10^{-4}\,\text{m}}$$

9. (II) A parallel beam of light from a He–Ne laser, with a wavelength 633 nm, falls on two very narrow slits 0.068 mm apart. How far apart are the fringes in the center of the pattern on a screen 3.8 m away?

For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = l \tan\theta$,

For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{l} = m\lambda \rightarrow x = \frac{\lambda m l}{d} \rightarrow$$

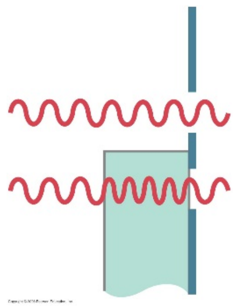
$$\Delta x = \Delta m\frac{\lambda l}{d} = (1)\frac{(633\times10^{-9}\,\text{m})(3.8\,\text{m})}{(6.8\times10^{-5}\,\text{m})} = 0.035\,\text{m} = \boxed{3.5\,\text{cm}}$$

10. (II) A physics professor wants to perform a lecture demonstration of Young's double-slit experiment for her class using the 633-nm light from a He–Ne laser. Because the lecture hall is very large, the interference pattern will be projected on a wall that is 5.0 m from the slits. For easy viewing by all students in the class, the professor wants the distance between the $m = 0$ and $m = 1$ maxima to be 25 cm. What slit separation is required in order to produce the desired interference pattern?

10. For constructive interference, the path difference is a multiple of the wavelength. The location on the screen is given by $x = l \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{l} = m\lambda \rightarrow d = \frac{\lambda m l}{x} = \frac{(633\times10^{-9}\,\text{m})(1)(5.0\,\text{m})}{(0.25\,\text{m})} = \boxed{1.3\times10^{-5}\,\text{m}}$$

11. (II) Suppose a thin piece of glass is placed in front of the lower slit in Fig. 34–7 so that the two waves enter the slits 180° out of phase (Fig. 34–25). Describe in detail the interference pattern on the screen.



11. The 180° phase shift produced by the glass is equivalent to a path length of $\frac{1}{2}\lambda$. For constructive interference on the screen, the total path difference is a multiple of the wavelength:

$$\tfrac{1}{2}\lambda + d\sin\theta_{max} = m\lambda,\ m = 0, 1, 2, \ldots \quad \rightarrow \quad d\sin\theta_{max} = \left(m - \tfrac{1}{2}\right)\lambda,\ m = 1, 2, \ldots$$

We could express the result as $d\sin\theta_{max} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \ldots$.

For destructive interference on the screen, the total path difference is

$$\tfrac{1}{2}\lambda + d\sin\theta_{min} = \left(m + \tfrac{1}{2}\right)\lambda,\ m = 0, 1, 2, \ldots \quad \rightarrow \quad d\sin\theta_{min} = m\lambda,\ m = 0, 1, 2, \ldots$$

Thus the pattern is just the $\boxed{\text{reverse of the usual double-slit pattern}}$. There will be a dark central line. Every place there was a bright fringe will now have a dark line, and vice versa.

12. (II) In a double-slit experiment it is found that blue light of wavelength 480 nm gives a second-order maximum at a certain location on the screen. What wavelength of visible light would have a minimum at the same location?

12. We equate the expression from Eq. 34-2a for the second order blue light to Eq. 34-2b, since the slit separation and angle must be the same for the two conditions to be met at the same location.

$$d\sin\theta = m\lambda_b = (2)(480\,\text{nm}) = 960\,\text{nm} \ ; \ d\sin\theta = \left(m' + \tfrac{1}{2}\right)\lambda,\ m' = 0, 1, 2, \ldots$$

$$\left(m' + \tfrac{1}{2}\right)\lambda = 960\,\text{nm} \quad m' = 0 \ \rightarrow \ \lambda = 1920\,\text{nm} \ ; \ m' = 1 \ \rightarrow \ \lambda = 640\,\text{nm}$$

$$m' = 2 \ \rightarrow \ \lambda = 384\,\text{nm}$$

The only one visible is $\boxed{640\text{ nm}}$. 384 nm is near the low-wavelength limit for visible light.

13. (II) Two narrow slits separated by 1.0 mm are illuminated by 544 nm light. Find the distance between adjacent bright fringes on a screen 5.0 m from the slits.

13. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = l \tan\theta$, For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$. For adjacent fringes, $\Delta m = 1$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{l} = m\lambda \rightarrow x = \frac{\lambda m l}{d} \rightarrow$$

$$\Delta x = \Delta m\frac{\lambda l}{d} = (1)\frac{\left(544\times10^{-9}\,\text{m}\right)\left(5.0\,\text{m}\right)}{\left(1.0\times10^{-3}\,\text{m}\right)} = \boxed{2.7\times10^{-3}\,\text{m}}$$

14. (II) In a double-slit experiment, the third-order maximum for light of wavelength 500 nm is located 12 mm from the central bright spot on a screen 1.6 m from the slits. Light of wavelength 650 nm is then projected through the same slits. How far from the central bright spot will the second-order maximum of this light be located?

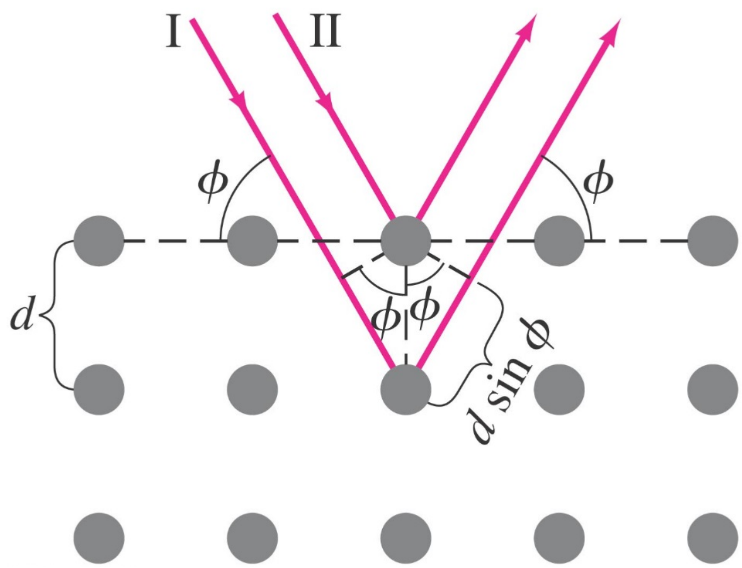
14. An expression is derived for the slit separation from the data for the 500 nm light. That expression is then used to find the location of the maxima for the 650 nm light. For constructive interference, the path difference is a multiple of the wavelength, as given by Eq. 34-2a. The location on the screen is given by $x = l \tan\theta$, as seen in Fig. 34-7(c). For small angles, we have $\sin\theta \approx \tan\theta \approx x/l$.

$$d\sin\theta = m\lambda \rightarrow d\frac{x}{l} = m\lambda \rightarrow d = \frac{\lambda m l}{x} = \frac{\lambda_1 m_1 l}{x_1} \rightarrow x = \frac{\lambda m l}{d} \rightarrow$$

$$x_2 = \frac{\lambda_2 m_2 l}{\frac{\lambda_1 m_1 l}{x_1}} = x_1\frac{\lambda_2 m_2}{\lambda_1 m_1} = (12\,\text{mm})\frac{(650\,\text{nm})(2)}{(500\,\text{nm})(3)} = 10.4\,\text{mm} \approx \boxed{10\,\text{mm}}\ \text{(2 sig. fig.)}$$

X-Ray Diffraction

49. (II) X-rays of wavelength 0.138 nm fall on a crystal whose atoms, lying in planes, are spaced 0.285 nm apart. At what angle θ (relative to the surface, Fig. below) must the X-rays be directed if the first diffraction maximum is to be observed?



49. We use Eq. 35-20, with $m = 1$.

$$m\lambda = 2d\sin\phi \rightarrow \phi = \sin^{-1}\frac{m\lambda}{2d} = \sin^{-1}\frac{(1)(0.138\,\text{nm})}{2(0.285\,\text{nm})} = \boxed{14.0^\circ}$$

50. (II) First-order Bragg diffraction is observed at 26.8° relative to the crystal surface, with spacing between atoms of 0.24 nm. (*a*) At what angle will second order be observed? (*b*) What is the wavelength of the X-rays?

50. We use Eq. 35-20 for X-ray diffraction.

(*a*) Apply Eq. 35-20 to both orders of diffraction.

$$m\lambda = 2d\sin\phi \rightarrow \frac{m_1}{m_2} = \frac{\sin\phi_1}{\sin\phi_2} \rightarrow \phi_2 = \sin^{-1}\left(\frac{m_2}{m_1}\sin\phi_1\right) = \sin^{-1}\left(\frac{2}{1}\sin 26.8^\circ\right) = \boxed{64.4^\circ}$$

(*b*) Use the first order data.

$$m\lambda = 2d\sin\phi \rightarrow \lambda = \frac{2d\sin\phi}{m} = \frac{2(0.24\,\text{nm})\sin 26.8^\circ}{1} = \boxed{0.22\,\text{nm}}$$

51. (II) If X-ray diffraction peaks corresponding to the first three orders ($m = 1$, 2, and 3) are measured, can both the X-ray wavelength λ and lattice spacing d be determined? Prove your answer.

51. For each diffraction peak, we can measure the angle and count the order. Consider Eq. 35-20.

$$m\lambda = 2d\sin\phi \rightarrow \lambda = 2d\sin\phi_1 \;;\; 2\lambda = 2d\sin\phi_2 \;;\; 3\lambda = 2d\sin\phi_3$$

From each equation, all we can find is the ratio $\frac{\lambda}{d} = 2\sin\phi = \sin\phi_2 = \frac{2}{3}\sin\phi_3$. $\boxed{\text{No}}$, we cannot separately determine the wavelength or the spacing.

Photons and the Photoelectric Effect

6. (I) What is the energy of photons (in joules) emitted by a 104.1-MHz FM radio station?

6. We use Eq. 37-3.

$$E = hf = \left(6.626\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(104.1\times10^{6}\,\text{Hz}\right) = \boxed{6.898\times10^{-26}\,\text{J}}$$

7. (I) What is the energy range (in joules and eV) of photons in the visible spectrum, of wavelength 410 nm to 750 nm?

$\boxed{7.}$ We use $f = c/\lambda$ for light. The longest wavelength will have the lowest energy.

$$E_1 = hf_1 = \frac{hc}{\lambda_1} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(410\times10^{-9}\,\text{m}\right)} = 4.85\times10^{-19}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 3.03\,\text{eV}$$

$$E_2 = hf_2 = \frac{hc}{\lambda_2} = \frac{\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(750\times10^{-9}\,\text{m}\right)} = 2.65\times10^{-19}\,\text{J}\left(\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}}\right) = 1.66\,\text{eV}$$

Thus the range of energies is $\boxed{2.7\times10^{-19}\,\text{J} < E < 4.9\times10^{-19}\,\text{J}}$ or $\boxed{1.7\,\text{eV} < E < 3.0\,\text{eV}}$.

12. (I) What is the longest wavelength of light that will emit electrons from a metal whose work function is 3.70 eV?

12. The longest wavelength corresponds to the minimum frequency. That occurs when the kinetic energy of the ejected electrons is 0.

$$K = hf_{\min} - W_0 = 0 \quad\rightarrow\quad f_{\min} = \frac{c}{\lambda_{\max}} = \frac{W_0}{h} \quad\rightarrow$$

$$\lambda_{\max} = \frac{ch}{W_0} = \frac{\left(3.00\times10^{8}\,\text{m/s}\right)\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)}{\left(3.70\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)} = \boxed{3.36\times10^{-7}\,\text{m}} = 336\,\text{nm}$$

13. (II) What wavelength photon would have the same energy as a 145-gram baseball moving 30.0 m/s?

$\boxed{13.}$ The energy of the photon will equal the kinetic energy of the baseball. We use Eq. 37-3.

$$K = hf \quad\rightarrow\quad \tfrac{1}{2}mv^2 = h\frac{c}{\lambda} \quad\rightarrow\quad \lambda = \frac{2hc}{mv^2} = \frac{2\left(6.63\times10^{-34}\,\text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\,\text{m/s}\right)}{\left(0.145\,\text{kg}\right)\left(30.0\,\text{m/s}\right)^2} = \boxed{3.05\times10^{-27}\,\text{m}}$$

14. (II) The human eye can respond to as little as $10^{-18} J$ of light energy. For a wavelength at the peak of visual sensitivity, 550 nm, how many photons lead to an observable flash?

14. We divide the minimum energy by the photon energy at 550 nm to find the number of photons.

$$E = nhf = E_{min} \quad \rightarrow \quad n = \frac{E_{min}}{hf} = \frac{E_{min}\lambda}{hc} = \frac{(10^{-18}\,\text{J})(550 \times 10^{-9}\,\text{m})}{(6.63 \times 10^{-34}\,\text{J}\cdot\text{s})(3.00 \times 10^{8}\,\text{m/s})} = 2.77 \approx \boxed{3 \text{ photons}}$$

15.(II) The work functions for sodium, cesium, copper, and iron are 2.3, 2.1, 4.7, and 4.5eV, respectively. Which of these metals will not emit electrons when visible light shines on it?

15. The photon of visible light with the maximum energy has the least wavelength. We use 410 nm as the lowest wavelength of visible light.

$$hf_{max} = \frac{hc}{\lambda_{min}} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(3.00\times10^{8}\,\text{m/s})}{(1.60\times10^{-19}\,\text{J/eV})(410\times10^{-9}\,\text{m})} = 3.03\,\text{eV}$$

Electrons will not be emitted if this energy is less than the work function.
The metals with work functions greater than 3.03 eV are $\boxed{\text{copper and iron.}}$

16.(II) In a photoelectric-effect experiment it is observed that no current flows unless the wavelength is less than 520 nm. (*a*) What is the work function of this material? (*b*) What is the stopping voltage required if light of wavelength 470 nm is used?

(*a*) At the threshold wavelength, the kinetic energy of the photoelectrons is zero, so the work function is equal to the energy of the photon.

$$W_0 = hf - K_{max} = hf = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{520\,\text{nm}} = \boxed{2.4\,\text{eV}}$$

(*b*) The stopping voltage is the voltage that gives a potential energy change equal to the maximum

kinetic energy. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV}\cdot\text{nm}}{470\,\text{nm}} - 2.38\,\text{eV} = 0.25\,\text{eV}$$

$$V_0 = \frac{K_{max}}{e} = \frac{0.25\,\text{eV}}{e} = \boxed{0.25\,\text{V}}$$

17.(II) What is the maximum kinetic energy of electrons ejected from barium ($\phi = 2.48eV$) when illuminated by white light, λ=410 to 750 nm?

17. The photon of visible light with the maximum energy has the minimum wavelength. We use Eq. 37-4b to calculate the maximum kinetic energy.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV}\cdot\text{nm}}{410\text{ nm}} - 2.48\text{ eV} = \boxed{0.54\text{ eV}}$$

18.(II) Barium has a work function of 2.48 eV. What is the maximum kinetic energy of electrons if the metal is illuminated by UV light of wavelength 365 nm? What is their speed?

18. We use Eq. 37-4b to calculate the maximum kinetic energy. Since the kinetic energy is much less than the rest energy, we use the classical definition of kinetic energy to calculate the speed.

$$K_{max} = hf - W_0 = \frac{hc}{\lambda} - W_0 = \frac{1240\,\text{eV·nm}}{365\,\text{nm}} - 2.48\,\text{eV} = \boxed{0.92\,\text{eV}}$$

$$K_{max} = \tfrac{1}{2}mv^2 \quad\rightarrow\quad v = \sqrt{\frac{2K_{max}}{m}} = \sqrt{\frac{2(0.92\,\text{eV})(1.60\times10^{-19}\,\text{J/eV})}{9.11\times10^{-31}\,\text{kg}}} = \boxed{5.7\times10^5\,\text{m/s}}$$

19. (II) When UV light of wavelength 285 nm falls on a metal surface, the maximum kinetic energy of emitted electrons is 1.70 eV. What is the work function of the metal?

19. We use Eq. 37-4b to calculate the work function.

$$W_0 = hf - K_{max} = \frac{hc}{\lambda} - K_{max} = \frac{1240\text{ eV·nm}}{285\text{ nm}} - 1.70\text{ eV} = \boxed{2.65\text{ eV}}$$

20. (II) The threshold wavelength for emission of electrons from a given surface is 320 nm. What will be the maximum kinetic energy of ejected electrons when the wavelength is changed to (*a*) 280 nm, (*b*) 360 nm?

20. Electrons emitted from photons at the threshold wavelength have no kinetic energy. We use Eq. 37-4b with the threshold wavelength to determine the work function.

$$W_0 = \frac{hc}{\lambda} - K_{max} = \frac{hc}{\lambda_{max}} = \frac{1240\text{ eV·nm}}{320\text{ nm}} = 3.88\text{ eV}.$$

(*a*) We now use Eq. 36-4b with the work function determined above to calculate the kinetic energy of the photoelectrons emitted by 280 nm light.

$$K_{max} = \frac{hc}{\lambda} - W_0 = \frac{1240\text{ eV·nm}}{280\text{ nm}} - 3.88\text{ eV} = \boxed{0.55\text{ eV}}$$

(*b*) Because the wavelength is greater than the threshold wavelength, the photon energy is less than the work function, so there will be $\boxed{\text{no ejected electrons.}}$

21. (II) When 230-nm light falls on a metal, the current through a photoelectric circuit is brought to zero at a stopping voltage of 1.84 V. What is the work function of the metal?

21. The stopping voltage is the voltage that gives a potential energy change equal to the maximum kinetic energy of the photoelectrons. We use Eq. 37-4b to calculate the work function where the maximum kinetic energy is the product of the stopping voltage and electron charge.

$$W_0 = \frac{hc}{\lambda} - K_{max} = \frac{hc}{\lambda} - eV_0 = \frac{1240\text{ eV·nm}}{230\text{ nm}} - (1.84\text{ V})e = \boxed{3.55\text{ eV}}$$

22. (II) A certain type of film is sensitive only to light whose wavelength is less than 630 nm. What is the energy (eV and kcal/mol) needed for the chemical reaction to occur which causes the film to change?

22. The energy required for the chemical reaction is provided by the photon. We use Eq. 37-3 for the energy of the photon, where $f = c/\lambda$.

$$E = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{630 \text{ nm}} = \boxed{2.0 \text{ eV}}$$

Each reaction takes place in a molecule, so we use the appropriate conversions to convert eV/molecule to kcal/mol.

$$E = \left(\frac{2.0 \text{ eV}}{\text{molecule}}\right)\left(\frac{1.60\times10^{-19} \text{ J}}{\text{eV}}\right)\left(\frac{6.02\times10^{23} \text{ molecules}}{\text{mol}}\right)\left(\frac{\text{kcal}}{4186 \text{ J}}\right) = \boxed{45 \text{ kcal/mole}}$$

23. (II) The range of visible light wavelengths extends from about 410 nm to 750 nm. (*a*) Estimate the minimum energy (eV) necessary to initiate the chemical process on the retina that is responsible for vision. (*b*) Speculate as to why, at the other end of the visible range, there is a threshold photon energy beyond which the eye registers no sensation of sight. Determine this threshold photon energy (eV).

23. (*a*) Since $f = c/\lambda$, the photon energy given by Eq. 37-3 can be written in terms of the wavelength as $E = hc/\lambda$. This shows that the photon with the largest wavelength has the smallest energy. The 750-nm photon then delivers the minimum energy that will excite the retina.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(750\times10^{-9} \text{ m})}\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right) = \boxed{1.66 \text{ eV}}$$

(*b*) The eye cannot see light with wavelengths less than 410 nm. Obviously, these wavelength photons have more energy than the minimum required to initiate vision, so they must not arrive at the retina. That is, wavelength less than 410 nm are absorbed near the front portion of the eye. The threshold photon energy is that of a 410-nm photon.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(410\times10^{-9} \text{ m})}\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right) = \boxed{3.03 \text{ eV}}$$

25. (II) A **photomultiplier tube** (a very sensitive light sensor), is based on the photoelectric effect: incident photons strike a metal surface and the resulting ejected electrons are collected. By counting the number of collected electrons, the number of incident photons (i.e., the incident light intensity) can be determined. (*a*) If a photomultiplier tube is to respond properly for incident wavelengths throughout the visible range (410 nm to 750 nm), what is the maximum value for the work function W_0 (eV) of its metal surface? (*b*) If W_0 for its metal surface is above a certain threshold value, the photomultiplier will only function for incident ultraviolet wavelengths and be unresponsive to visible light. Determine this threshold value (eV).

25. (*a*) Since $f = c/\lambda$, the photon energy is $E = hc/\lambda$ and the largest wavelength has the smallest energy. In order to eject electrons for all possible incident visible light, the metal's work function must be less than or equal to the energy of a 750-nm photon. Thus the maximum value for the metal's work function W_o is found by setting the work function equal to the energy of the 750-nm photon.

$$W_o = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\,\mathrm{J{\cdot}s})(3.00\times10^{8}\,\mathrm{m/s})}{(750\times10^{-9}\,\mathrm{m})}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right) = \boxed{1.66\,\mathrm{eV}}$$

(*b*) If the photomultiplier is to function only for incident wavelengths less than 410-nm, then we set the work function equal to the energy of the 410-nm photon.

$$W_o = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\,\mathrm{J{\cdot}s})(3.00\times10^{8}\,\mathrm{m/s})}{(410\times10^{-9}\,\mathrm{m})}\left(\frac{1\,\mathrm{eV}}{1.60\times10^{-19}\,\mathrm{J}}\right) = \boxed{3.03\,\mathrm{eV}}$$

Compton Effect

28. (I) A high-frequency photon is scattered off of an electron and experiences a change of wavelength of 1.5 X 10^{-4} nm . At what angle must a detector be placed to detect the scattered photon (relative to the direction of the incoming photon)?

28. We use Eq. 37-6b. Note that the answer is correct to two significant figures.

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\phi) \quad \rightarrow$$

$$\phi = \cos^{-1}\left(1-\frac{m_e c\Delta\lambda}{h}\right) = \cos^{-1}\left(1-\frac{(9.11\times10^{-31}\,\mathrm{kg})(3.00\times10^{8}\,\mathrm{m/s})(1.5\times10^{-13}\,\mathrm{m})}{(6.63\times10^{-34}\,\mathrm{J{\cdot}s})}\right) = \boxed{20^\circ}$$

29. (II) Determine the Compton wavelength for (*a*) an electron, (*b*) a proton. (*c*) Show that if a photon has wavelength equal to the Compton wavelength of a particle, the photon's energy is equal to the rest energy of the particle.

29. The Compton wavelength for a particle of mass m is h/mc.

(*a*) $$\frac{h}{m_e c} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{(9.11\times10^{-31}\,\text{kg})(3.00\times10^{8}\,\text{m/s})} = \boxed{2.43\times10^{-12}\,\text{m}}$$

(*b*) $$\frac{h}{m_p c} = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})}{(1.67\times10^{-27}\,\text{kg})(3.00\times10^{8}\,\text{m/s})} = \boxed{1.32\times10^{-15}\,\text{m}}$$

(*c*) The energy of the photon is given by Eq. 37-3.

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{hc}{(h/mc)} = mc^2 = \text{rest energy}$$

30. (II) X-rays of wavelength λ=0.120nm are scattered from carbon. What is the expected Compton wavelength shift for photons detected at angles (relative to the incident beam) of exactly (*a*) 60°, (*b*) 90°, (*c*) 180°?

30. We find the Compton wavelength shift for a photon scattered from an electron, using Eq. 37-6b. The Compton wavelength of a free electron is given in the text right after Eq. 37-6b.

$$\lambda' - \lambda = \left(\frac{h}{m_e c}\right)(1-\cos\theta) = \lambda_C(1-\cos\theta) = (2.43\times10^{-3}\,\text{nm})(1-\cos\theta)$$

(*a*) $$\lambda'_a - \lambda = (2.43\times10^{-3}\,\text{nm})(1-\cos 60^\circ) = \boxed{1.22\times10^{-3}\,\text{nm}}$$

(*b*) $$\lambda'_b - \lambda = (2.43\times10^{-3}\,\text{nm})(1-\cos 90^\circ) = \boxed{2.43\times10^{-3}\,\text{nm}}$$

(*c*) $$\lambda'_c - \lambda = (2.43\times10^{-3}\,\text{nm})(1-\cos 180^\circ) = \boxed{4.86\times10^{-3}\,\text{nm}}$$

31. (II) In the Compton effect, determine the ratio (Δλ/λ) of the maximum change Δλ in a photon's wavelength to the photon's initial wavelength λ, if the photon is (*a*) a visible-light photon with λ=550nm (*b*) an X-ray photon with λ=0.10nm.

31. (*a*) In the Compton effect, the maximum change in the photon's wavelength is when scattering angle $\phi = 180^\circ$. We use Eq. 37-6b to determine the maximum change in wavelength. Dividing the maximum change by the initial wavelength gives the maximum fractional change.

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) \rightarrow$$

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_e c\lambda}(1-\cos\theta) = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(1-\cos180^\circ)}{(9.11\times10^{-31}\,\text{kg})(3.00\times10^{8}\,\text{m/s})(550\times10^{-9}\,\text{m})} = \boxed{8.8\times10^{-6}}$$

(*b*) We replace the initial wavelength with $\lambda = 0.10\ \text{nm}$.

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{m_e c\lambda}(1-\cos\theta) = \frac{(6.63\times10^{-34}\,\text{J}\cdot\text{s})(1-\cos180^\circ)}{(9.11\times10^{-31}\,\text{kg})(3.00\times10^{8}\,\text{m/s})(0.10\times10^{-9}\,\text{m})} = \boxed{0.049}$$

32. (II) A 1.0-MeV gamma-ray photon undergoes a sequence of Compton-scattering events. If the photon is scattered at an angle of 0.50° in each event, estimate the number of events required to convert the photon into a visible-light photon with wavelength 555 nm. You can use an expansion for small θ; [Gamma rays created near the center of the Sun are transformed to visible wavelengths as they travel to the Sun's surface through a sequence of small-angle Compton scattering events.]

32. We find the change in wavelength for each scattering event using Eq. 37-6b, with a scattering angle of $\phi = 0.50^\circ$. To calculate the total change in wavelength, we subtract the initial wavelength, obtained from the initial energy, from the final wavelength. We divide the change in wavelength by the wavelength change from each event to determine the number of scattering events.

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos0.5^\circ) = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(1-\cos0.5^\circ)}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = 9.24\times10^{-17}\text{m} = 9.24\times10^{-8}\text{nm}$$

$$\lambda_0 = \frac{hc}{E_0} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.0\times10^{6}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 1.24\times10^{-12}\text{m} = 0.00124\,\text{nm}.$$

$$n = \frac{\lambda-\lambda_0}{\Delta\lambda} = \frac{(555\ \text{nm})-(0.00124\ \text{nm})}{9.24\times10^{-8}\ \text{nm}} = \boxed{6\times10^{9}\,\text{events}}$$

33. (III) In the Compton effect, a 0.160-nm photon strikes a free electron in a head-on collision and knocks it into the forward direction. The rebounding photon recoils directly backward. Use conservation of (relativistic) energy and momentum to determine (*a*) the kinetic energy of the electron, and (*b*) the wavelength of the recoiling photon.

33. (*a*) We use conservation of momentum to set the initial momentum of the photon equal to the sum

of the final momentum of the photon and electron, where the momentum of the photon is given by Eq. 37-5 and the momentum of the electron is written in terms of the total energy (Eq. 36-13). We multiply this equation by the speed of light to simplify.

$$\frac{h}{\lambda} + 0 = -\left(\frac{h}{\lambda'}\right) + p_e \quad\rightarrow\quad \frac{hc}{\lambda} = -\left(\frac{hc}{\lambda'}\right) + \sqrt{E^2 - E_0^2}$$

Using conservation of energy we set the initial energy of the photon and rest energy of the electron equal to the sum of the final energy of the photon and the total energy of the electron.

$$\left(\frac{hc}{\lambda}\right) + E_0 = \left(\frac{hc}{\lambda'}\right) + E$$

By summing these two equations, we eliminate the final wavelength of the photon. We then solve the resulting equation for the kinetic energy of the electron, which is the total energy less the rest energy.

$$2\left(\frac{hc}{\lambda}\right)+E_0=\sqrt{E^2-E_0^2}+E \quad\to\quad \left[2\left(\frac{hc}{\lambda}\right)+E_0-E\right]^2=E^2-E_0^2$$

$$\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2-2E\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]+E^2=E^2-E_0^2 \quad\to\quad E=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^2}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$K=E-E_0=\frac{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]^2+E_0^2}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}-\frac{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]E_0}{2\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}=\frac{2\left(\frac{hc}{\lambda}\right)^2}{\left[2\left(\frac{hc}{\lambda}\right)+E_0\right]}$$

$$=\frac{2\left(\frac{1240\text{ eV}\cdot\text{nm}}{0.160\text{ nm}}\right)^2}{\left[2\left(\frac{1240\text{ eV}\cdot\text{nm}}{0.160\text{ nm}}\right)+5.11\times10^5\text{eV}\right]}=\boxed{228\text{eV}}$$

(*b*) We solve the energy equation for the final wavelength.

$$\left(\frac{hc}{\lambda}\right)+E_0=\left(\frac{hc}{\lambda'}\right)+E$$

$$\lambda'=\frac{hc}{\left(\frac{hc}{\lambda}\right)+E_0-E}=\left[\frac{1}{\lambda}-\frac{K}{hc}\right]^{-1}=\left[\frac{1}{0.160\text{nm}}-\frac{228\text{eV}}{1240\text{eV}\cdot\text{nm}}\right]^{-1}=\boxed{0.165\text{nm}}$$

Pair Production

35. (I) How much total kinetic energy will an electron–positron pair have if produced by a 2.67-MeV photon?

35. The photon energy must be equal to the kinetic energy of the products plus the mass energy of the products. The mass of the positron is equal to the mass of the electron.

$$E_{\text{photon}}=K_{\text{products}}+m_{\text{products}}c^2 \quad\to$$

$$K_{\text{products}}=E_{\text{photon}}-m_{\text{products}}c^2=E_{\text{photon}}-2m_{\text{electron}}c^2=2.67\text{MeV}-2(0.511\text{MeV})=\boxed{1.65\text{MeV}}$$

36. (II) What is the longest wavelength photon that could produce a proton–antiproton pair? (Each has a mass of $1.67\text{X}10^{-27}$kg)

36. The photon with the longest wavelength has the minimum energy in order to create the masses with no additional kinetic energy. Use Eq. 37-5.

$$\lambda_{max} = \frac{hc}{E_{min}} = \frac{hc}{2mc^2} = \frac{h}{2mc} = \frac{(6.63 \times 10^{-34}\,\mathrm{J{\cdot}s})}{2(1.67 \times 10^{-27}\,\mathrm{kg})(3.00 \times 10^8\,\mathrm{m/s})} = \boxed{6.62 \times 10^{-16}\,\mathrm{m}}$$

This must take place in the presence of some other object in order for momentum to be conserved.

37. (II) What is the minimum photon energy needed to produce a $\mu^+ - \mu^-$ pair? The mass of each μ (muon) is 207 times the mass of an electron. What is the wavelength of such a photon?

37. The minimum energy necessary is equal to the rest energy of the two muons.

$$E_{min} = 2mc^2 = 2(207)(0.511\,\mathrm{MeV}) = \boxed{212\,\mathrm{MeV}}$$

The wavelength is given by Eq. 37-5.

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\,\mathrm{J{\cdot}s})(3.00 \times 10^8\,\mathrm{m/s})}{(1.60 \times 10^{-19}\,\mathrm{J/eV})(212 \times 10^6\,\mathrm{eV})} = \boxed{5.86 \times 10^{-15}\,\mathrm{m}}$$

38. (II) An electron and a positron, each moving at $2.0 \mathrm{X} 10^5$m/s collide head on, disappear, and produce two photons moving in opposite directions, each with the same energy and momentum. Determine the energy and momentum of each photon.

38. Since $v < 0.001c$, the total energy of the particles is essentially equal to their rest energy. Both particles have the same rest energy of 0.511 MeV. Since the total momentum is 0, each photon must have half the available energy and equal momenta.

$$E_{photon} = m_{electron}c^2 = \boxed{0.511\,\mathrm{MeV}} \quad ; \quad p_{photon} = \frac{E_{photon}}{c} = \boxed{0.511\,\mathrm{MeV}/c}$$

39. (II) A gamma-ray photon produces an electron and a positron, each with a kinetic energy of 375 keV. Determine the energy and wavelength of the photon.

39. The energy of the photon is equal to the total energy of the two particles produced. Both particles have the same kinetic energy and the same mass.

$$E_{photon} = 2(K + mc^2) = 2(0.375\,\mathrm{MeV} + 0.511\,\mathrm{MeV}) = \boxed{1.772\,\mathrm{MeV}}$$

The wavelength is found from Eq. 37-5.

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}\,\mathrm{J{\cdot}s})(3.00 \times 10^8\,\mathrm{m/s})}{(1.60 \times 10^{-19}\,\mathrm{J/eV})(1.772 \times 10^6\,\mathrm{eV})} = \boxed{7.02 \times 10^{-13}\,\mathrm{m}}$$

Wave Nature of Matter

40. Calculate the wavelength of a 0.23-kg ball traveling at 0.140m/s

40. We find the wavelength from Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)}{\left(0.23\,\mathrm{kg}\right)\left(0.10\,\mathrm{m/s}\right)} = \boxed{2.9\times10^{-32}\,\mathrm{m}}$$

41. What is the wavelength of a neutron traveling at 8.5×10^{4}m/s

41.The neutron is not relativistic, so we can use $p = mv$. We also use Eq. 37-7.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)}{\left(1.67\times10^{-27}\,\mathrm{kg}\right)\left(8.5\times10^{4}\,\mathrm{m/s}\right)} = \boxed{4.7\times10^{-12}\,\mathrm{m}}$$

42. Through how many volts of potential difference must an electron be accelerated to achieve a wavelength of 0.21 nm?

We assume the electron is non-relativistic, and check that with the final answer. We use

$$\lambda = \frac{h}{p} = \frac{h}{mv} \rightarrow v = \frac{h}{m\lambda} = \frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(0.21\times10^{-9}\,\mathrm{m}\right)} = 3.466\times10^{6}\,\mathrm{m/s} = 0.01155c$$

Our use of classical expressions is justified. The kinetic energy is equal to the potential energy change.

$$eV = K = \tfrac{1}{2}mv^2 = \frac{\tfrac{1}{2}\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(3.466\times10^{6}\,\mathrm{m/s}\right)^2}{\left(1.60\times10^{-19}\,\mathrm{J/eV}\right)} = 34.2\,\mathrm{eV}$$

Thus the required potential difference is $\boxed{34\ \mathrm{V.}}$

44. The speed of an electron in a particle accelerator is 0.98c. Find its de Broglie wavelength. (Use relativistic momentum.)

44. We use the relativistic expression for momentum, Eq. 36-8.

$$p = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{h}{\lambda} \rightarrow$$

$$\lambda = \frac{h\sqrt{1-v^2/c^2}}{mv} = \frac{\left(6.63\times10^{-34}\,\mathrm{J\cdot s}\right)\sqrt{1-\left(0.98\right)^2}}{\left(9.11\times10^{-31}\,\mathrm{kg}\right)\left(0.98\right)\left(3.00\times10^{8}\,\mathrm{m/s}\right)} = \boxed{4.9\times10^{-13}\,\mathrm{m}}$$

47. An electron has a de Broglie wavelength (a) What is its momentum? (b) What is its speed? (c) What voltage was needed to accelerate it to this speed?

47.(*a*) We find the momentum from Eq. 37-7.

$$p=\frac{h}{\lambda}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{6.0\times10^{-10}\,\text{m}}=\boxed{1.1\times10^{-24}\,\text{kg}\cdot\text{m/s}}$$

(*b*) We assume the speed is non-relativistic.

$$\lambda=\frac{h}{p}=\frac{h}{mv}\quad\rightarrow\quad v=\frac{h}{m\lambda}==\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{kg}\right)\left(6.0\times10^{-10}\text{m}\right)}=\boxed{1.2\times10^{6}\,\text{m/s}}$$

Since $v/c=4.04\times10^{-3}$, our assumption is valid.

(*c*) We calculate the kinetic energy classically.

$$K=\tfrac{1}{2}mv^2=\tfrac{1}{2}\left(mc^2\right)\left(v/c\right)^2=\tfrac{1}{2}\left(0.511\,\text{MeV}\right)\left(4.04\times10^{-3}\right)^2=4.17\times10^{-6}\,\text{MeV}=4.17\,\text{eV}$$

This is the energy gained by an electron if accelerated through a potential difference of $\boxed{4.2\text{ V}}$.

48.What is the wavelength of an electron of energy (a) 20 eV, (b) 200 eV, (c) 2.0 keV?

Because all of the energies to be considered are much less than the rest energy of an electron, we can use non-relativistic relationships. We use Eq. 37-7 to calculate the wavelength.

$$K=\frac{p^2}{2m}\quad\rightarrow\quad p=\sqrt{2mK}\ ;\ \lambda=\frac{h}{p}=\frac{h}{\sqrt{2mK}}$$

(*a*) $$\lambda=\frac{h}{\sqrt{2mK}}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(20\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}}=2.7\times10^{-10}\text{m}\approx\boxed{3\times10^{-10}\text{m}}$$

(*b*) $$\lambda=\frac{h}{\sqrt{2mK}}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(200\,\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}}=8.7\times10^{-11}\text{m}\approx\boxed{9\times10^{-11}\text{m}}$$

(*c*) $$\lambda=\frac{h}{\sqrt{2mK}}=\frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\text{kg}\right)\left(2.0\times10^{3}\text{eV}\right)\left(1.60\times10^{-19}\,\text{J/eV}\right)}}=\boxed{2.7\times10^{-11}\text{m}}$$