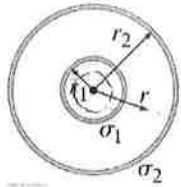


Problem 1 (33 Points)

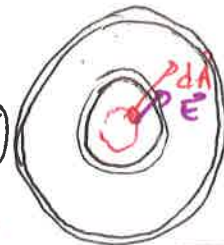
Two thin concentric spherical shells of radii r_1 and r_2 ($r_1 < r_2$) contain uniform surface charge densities σ_1 and σ_2 respectively (see Fig. below).



Gauss's Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{en}}{\epsilon_0}$ (3)

Determine the electric field for

(a) $0 < r < r_1$, $\oint \vec{E} \cdot d\vec{A} = \int E dA \cos \theta = E \int dA = E(4\pi r^2)$



(10) $Q_{en} = 0$ inside $\Rightarrow \oint \vec{E} \cdot d\vec{A} = \frac{Q_{en}}{\epsilon_0} = 0$
 $E(4\pi r^2) = 0 \Rightarrow E = 0$

$E = 0$

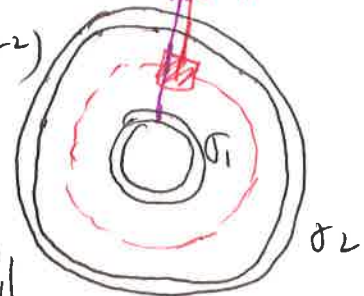
(b) $r_1 < r < r_2$,

$\oint \vec{E} \cdot d\vec{A} = \int E dA \cos \theta = E \int dA = E(4\pi r^2)$

$Q_{en} = \sigma_1 A_1 = \sigma_1 (4\pi r_1^2)$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{en}}{\epsilon_0}$

(10) $E(4\pi r^2) = \frac{\sigma_1 (4\pi r_1^2)}{\epsilon_0} \Rightarrow E = \frac{\sigma_1}{\epsilon_0} \frac{r_1^2}{r^2}$



and (c) $r > r_2$,

$\oint \vec{E} \cdot d\vec{A}$ stay the same $= E(4\pi r^2)$

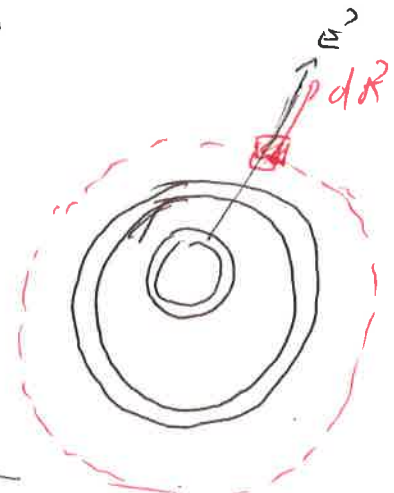
$Q_{en} = \sigma_1 A_1 + \sigma_2 A_2$

(10) $Q_{en} = \sigma_1 (4\pi r_1^2) + \sigma_2 (4\pi r_2^2)$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{en}}{\epsilon_0}$

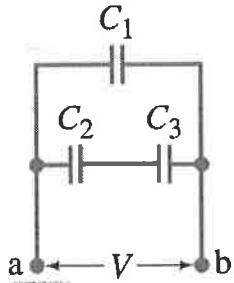
$E(4\pi r^2) = \frac{1}{\epsilon_0} (\sigma_1 (4\pi r_1^2) + \sigma_2 (4\pi r_2^2))$

$E = \frac{\sigma_1 r_1^2 + \sigma_2 r_2^2}{\epsilon_0 r^2}$



Problem 2 (34 Points)

(a) Determine the equivalent capacitance of the circuit shown in figure below.



$$C_2 \parallel C_3 \Rightarrow \frac{1}{C_{23}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_3 + C_2}{C_2 C_3}$$

$$\Rightarrow C_{23} = \frac{C_2 C_3}{C_3 + C_2} \quad (16)$$

$$C_{eq} = C_1 + \frac{C_2 C_3}{C_3 + C_2}$$

(b) If \$C_1=C_2=2C_3=24\mu\text{F}\$, how much charge is stored on each capacitor when \$V = 35.0\text{V}\$?

$$V_{ab} = V_{C_2 C_3} = V_{C_1} = 35.0\text{V}$$

$$Q_1 = C_1 V_{C_1} = (24 \times 10^{-6}\text{F})(35.0\text{V}) = 8.4 \times 10^{-4}\text{C}$$

$$Q_{23} = C_{23} V_{C_2 C_3}$$

$$C_{23} = \frac{C_2 C_3}{C_3 + C_2}$$

$$C_3 = \frac{1}{2} C_2 = \frac{1}{2} C \quad (18)$$

$$C_{23} = \frac{C(\frac{1}{2}C)}{\frac{1}{2}C + C} = \frac{\frac{1}{2}C^2}{\frac{1}{2}C + \frac{2C}{2}} = \frac{\frac{1}{2}C^2}{\frac{3}{2}C} = \frac{C}{3}$$

$$Q_{23} = \frac{C}{3} V_{C_2 C_3} = \frac{24 \times 10^{-6}\text{F}}{3} (35.0\text{V}) = 2.80 \times 10^{-4}\text{C}$$

$$= Q_2 = Q_3 \quad (\text{same charge } (C_2 \times C_3 \text{ in series}))$$

Problem 3 (33 Points)

An extension cord made of two wires of diameter 0.129 cm and of length 2.7 m is connected to an electric heater which draws 15.0 A on a 120-V line. The resistivity of the copper is $1.68 \times 10^{-8} \Omega \cdot m$.

How much power is dissipated in the cord?

$$\begin{aligned} V &= 120 \text{ V} \\ I &= 15.0 \text{ A} \\ l &= 2.7 \text{ m} \\ d &= 0.129 \text{ cm} \\ \rho &= 1.68 \times 10^{-8} \Omega \cdot m \end{aligned}$$

$$P = I^2 R \quad (5)$$

$$R = \frac{\rho l}{A} \quad (5)$$

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \quad (6)$$

$$P = I^2 \frac{\rho l}{A} = I^2 \frac{\rho l}{\frac{\pi d^2}{4}} = \frac{4 I^2 \rho l}{\pi d^2} \quad (8)$$

$$P = \frac{4 (15.0 \text{ A})^2 (1.68 \times 10^{-8} \Omega \cdot m) (5.4 \text{ m})}{\pi (0.129 \times 10^{-2} \text{ m})^2} = 15.62 \text{ W}$$

$l = 2 \times 2.7 \text{ m} = 5.4 \text{ m} \quad (4)$