Name Key

Problem 1: (33 points)

A particle is at the position $(x, y, z) = (1.0, 2.0, 3.0)$ m. It is traveling with a vector velocity $(-5.0, +2.8, -3.1)\ m/s$. Its mass is 3.8 kg. What is its **vector angular momentum** about the origin?

$\vec{r}$ (1.0, 2.0, 3.0) m

$\vec{v}$ = (−5.0, +2.8, −3.1) m/s (4) $m = 3.8$ kg

$\vec{L} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$ (2) (2)

$$= 3.8 \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.0 & 2.0 & 3.0 \\ -5.0 & +2.8 & -3.1 \end{vmatrix}$$ (8)

$= 3.8 \left[\hat{i}(-6.2 - 8.4) - \hat{j}(-3.1 + 15) + \hat{k}(2.8 - (-10))\right]$ (8)

$= 3.8 \left[\hat{i}(-14.6) - \hat{j}(+11.9) + \hat{k}(12.8)\right]$ (9)

$\vec{L} = -55.5\hat{i} - 45.2\hat{j} + 48.6\hat{k}$

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Problem 2: (34 Points)

A carousel is initially at rest. At t = 0 it is given a constant angular acceleration $\alpha = 0.040$ rad/s^2, which increases its angular velocity for 6.0 s. At t = 6.0 s, determine the magnitude of the following quantities:

(a) the angular velocity of the carousel;

$\omega = \alpha t + \omega_0$

$\omega = 0.040(6) + 0 = 0.24$ rad/s (8)

(b) the linear velocity of a child located 2.5 m from the center;

$v = R\omega = (2.5\text{m})(0.24\ \text{rad/s}) = 0.6$ m/s (9)

(c) the tangential (linear) acceleration of that child;

$a_t = r\alpha = (2.5\text{m})(0.04\ \text{rad/s}^2) = 0.1$ m/s^2 (8)

(d) the centripetal acceleration of the child;

$a_R = \frac{v^2}{R} = \frac{(0.6\text{m/s})^2}{2.5} = 0.144$ m/s^2 (9)

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Problem 3: (33 points)

A 1.15-kg mass oscillates according to the equation $x = 0.650\cos 7.40t$ where x is in meters and t in seconds. Determine

(a) the amplitude,

$A = 0.650\,m$ (9)

(b) the frequency,

$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{7.4}{2\pi} = 1.18\,Hz$ (8)

(c) the total energy,

$E = \frac{1}{2}kA^2$ with $\omega = \sqrt{\frac{k}{m}}$ so $\omega^2 = \frac{k}{m} \Rightarrow k = \omega^2 m$

$\mathcal{E} = \frac{1}{2}\omega^2 m A^2 = \frac{1}{2}(1.15kg)(7.40\,rad/s)^2(0.650m)^2 = 13.30\,J$ (8)

and (*d*) the kinetic energy and potential energy when $x = 0.260$ m.

$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}(1.15kg)\left(7.40\frac{rad}{s}\right)^2(0.260m)^2$

$= 2.1\,J$

$K = E - U = 13.30\,J - 2.1\,J$ (8)

$= 11.2\,J$