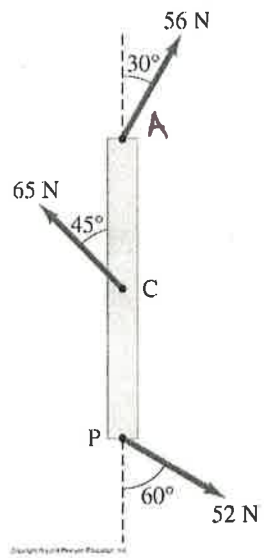
Name KEY

Problem 1: (33 Points)

Determine the net torque on the 2.0-m-long uniform beam shown in Fig. below.

(Hint: Assume that the force 52N is CCW)



Counter clockwise is positive

Calculate about:

(a) point C, the CM,

Each lever arm is 1.0 m for 56 N & 52 N. No torque for 65 N since it is at the center of mass

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$\tau_{about\ C} = -F_{56N}(AC)\sin 30 + F_{52N}(PC)\sin 60°$

$= (-56N)(1.0m)\sin 30 + (52N)(1.0m)\sin 60° = 17\ Nm$

$\boxed{\tau_{/C} = 17\ Nm}$

and (*b*) point P at one end.

τ of 52N is zero; lever arm = 0

$\tau_{about\ P} = -F_{56}(AP)\sin 30 + F_{65}(CP)\sin 60°$

$= -(56N)(2m)\sin 30 + (65N)(1.0m)\sin 60°$

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$\tau_{/P} = -10\ N.m$

$\boxed{\tau_{/P} = -10\ Nm}$

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Problem 2: (34 Points)

37.(II) A ball of mass 0.220 kg that is moving with a speed of 7.5m/s collides head-on and elastically with another ball initially at rest. Immediately after the collision, the incoming ball bounces backward with a speed of -3.8m/s. Calculate

(a) the velocity of the target ball after the collision,

6 $v_A - \cancel{v_B}^{0} = -(v_A' - v_B') \Rightarrow v_A = -v_A' + v_B' \Rightarrow v_B' = v_A' + v_A$ 4 4

$v_A' = -3.8\,m/s$

$v_A = 7.5\,m/s$

$v_B' = -3.8\,m/s + 7.5\,m/s = 3.7\,m/s$ 4

and (b) the mass of the target ball.

$m_A v_A + \cancel{m_B v_B}^{0} = m_A v_A' + m_B v_B'$ 4

4 $m_A v_A - m_A v_A' = m_B v_B' \Rightarrow m_B = \dfrac{m_A v_A - m_A v_A'}{v_B'}$ 4

$m_B = \dfrac{m_A(v_A - v_A')}{v_B'}$ 2 $= \dfrac{0.220\,kg(7.5\,m/s - (-3.8\,m/s))}{3.7\,m/s}$ 2 $=$

$m_B = 0.67\,kg$

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Problem 3: (33 Points)

(II) The angle through which a rotating wheel has turned in time t is given by

$\theta = 9t - 7.0t^2 + 2t^3$ where θ is in radians and t in seconds. Determine an expression

(a) for the instantaneous angular velocity ω and

$\omega = \frac{d\theta}{dt} = (9 - 14.0t + 6t^2)\ \text{rad/s}$

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(b) for the instantaneous angular acceleration α

$\alpha = \frac{d\omega}{dt} = 0 - 14.0 + 12.0t = (-14.0 + 12.0t)\ \text{rad/s}^2$

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(c) Evaluate ω and α at $t=3.0s$

$\omega(3.0s) = 9 - 14.0(3.0s) + 6(3.0s)^2 = 21\ \text{rad/s}$

$\alpha(3.0s) = -14.0 + 12.0(3.0s) = 22\ \text{rad/s}^2$

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(d) What is the average angular velocity, and the average angular acceleration between $t=2.0s$ and $t=3.0s$

$\omega_{avg} = \frac{\Delta\theta}{\Delta t} = \frac{\theta(3.0s) - \theta(2.0s)}{\underbrace{3.0s - 2.0s}_{=1s}} = \begin{matrix} 9.0 - 14(3.0s) + 6.0(3.0s)^2 \\ -[9.0 - 14(2.0s) + 6.0(2.0s)^2] \end{matrix}$

$= 12\ \text{rad/s}$

$\omega_{avg} = 16\ \text{rad/s}$

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$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega(3.0s) - \omega(2.0s)}{1s} = \frac{[-14.0 + 12(3.0s)] - [-14.0 + 12(2.0s)]}{1s}$

$\alpha_{avg} = 16\ \text{rad/s}^2$

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Bonus Problem: (10 Points)

What is **the dot and the cross products** of $\vec{A} = 2.0x^3\hat{\imath}-2.0x\hat{\jmath}+1.0\hat{k}$ and $\vec{B} = 11.0\hat{\imath}+2.5x\hat{\jmath}$?

$$\vec{A}\cdot\vec{B} = (2.0x^3\hat{\imath}-2.0x\hat{\jmath}+1.0\hat{k})\cdot(11.0\hat{\imath}+2.5x\hat{\jmath})$$

$$= (2.0x^3)(11.0) + (-2.0x)(2.5x) + (1.0)(0)$$

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$$\boxed{\vec{A}\cdot\vec{B} = 22.0x^3 - 5x^2}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2.0x^3 & -2.0x & 1.0 \\ 11.0 & 2.5x & 0 \end{vmatrix} = \hat{\imath}(0-2.5x) - \hat{\jmath}(0-11.0) + \hat{k}\left((2.0x^3)(2.5x) - (-2.0x)(11.0)\right)$$

$$= \hat{\imath}(-2.5x) - \hat{\jmath}(-11.0-0) + \hat{k}(5.0x^4 + 22.0x)$$

$$= (-2.5)x\hat{\imath} + (11.0)\hat{\jmath} + (5x^4 + 22.0x)\hat{k}$$

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