

Name KEY**Problem 1: (33 Points)**Three vectors $\vec{V}_1 = 4.0\hat{i} - 8.0\hat{j}$, $\vec{V}_2 = \hat{i} + \hat{j}$, and $\vec{V}_3 = -3.0\hat{i} + \hat{j}$ Determine the magnitude and direction of(a) \vec{V}_1 ,

$$|\vec{V}_1| = \sqrt{(4.0)^2 + (-8.0)^2} = 8.95 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{-8.0}{4.0}\right) = -63.43^\circ$$

8

(b) \vec{V}_2 ,

$$|\vec{V}_2| = \sqrt{(1.0)^2 + (1.0)^2} = 1.414 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$$

8

(c) \vec{V}_3 ,

$$|\vec{V}_3| = \sqrt{(-3.0)^2 + (1.0)^2} = 3.17 \text{ units}$$

$$\theta = \tan^{-1}\left(\frac{1.0}{-3.0}\right) = -18.41^\circ$$

8

(b) Determine the magnitude and direction of

$$\vec{V}_1 - 2\vec{V}_2 = (4.0\hat{i} - 8.0\hat{j}) - 2(\hat{i} + \hat{j}) = (4.0\hat{i} - 2.0\hat{i}) - 8.0\hat{j} - 2\hat{j}$$

$$\vec{V}_1 - 2\vec{V}_2 = 2.0\hat{i} - 10\hat{j}$$

$$|\vec{V}_1 - 2\vec{V}_2| = \sqrt{(2.0)^2 + (-10)^2} = 10.20 \text{ units}$$

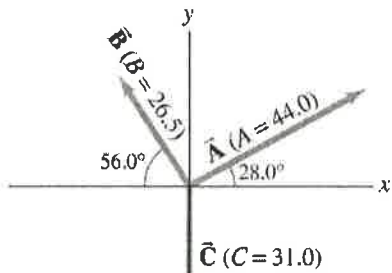
$$\theta = \tan^{-1}\left(\frac{-10}{2.0}\right) = -78.70^\circ$$

9

Problem 2: (34 Points)

(II) Three vectors are shown in figure below. Their magnitudes are given in arbitrary units.

(a) determine $\vec{B} - \vec{A}$.



$\vec{B} - \vec{A} = ?$

$$\vec{A} = \begin{cases} A_x = A \cos \theta = 44 \cos 28 = 38.85 \text{ units} \\ A_y = A \sin \theta = 44 \sin 28 = 20.66 \text{ units} \end{cases}$$

$$\vec{B} = \begin{cases} B_x = -B \cos \theta = -26.5 \cos 56 = -14.82 \text{ units} \\ B_y = B \sin \theta = 26.5 \sin 56 = 21.97 \text{ units} \end{cases}$$

$\vec{B} - \vec{A} = \begin{cases} B_x - A_x = -14.82 - 38.84 = -53.67 \text{ units} \\ B_y - A_y = 21.97 - 20.66 = 1.31 \text{ units} \end{cases}$

$$|\vec{B} - \vec{A}| = \sqrt{(-53.67)^2 + (1.31)^2} = 53.70 \text{ units}$$

$$\theta = \tan^{-1} \left(\frac{1.31}{-53.67} \right) = 1.4^\circ \text{ above } -x \text{ axis}$$

$\vec{B} - \vec{A} = (-53.67 \hat{i} + 1.31 \hat{j}) \text{ units}$

(b) Determine $\vec{A} - \vec{B}$ without using your answer in (a). Then compare your results and see if they are opposite.

$$\vec{A} - \vec{B} = \begin{cases} A_x - B_x = 38.85 - (-14.82) = 53.67 \text{ units} \\ A_y - B_y = 20.66 - 21.97 = -1.31 \text{ units} \end{cases}$$

$$|\vec{A} - \vec{B}| = \sqrt{(53.67)^2 + (-1.31)^2} = 53.7^\circ$$

$$\theta = \tan^{-1} \left(\frac{-1.31}{53.67} \right) = -1.4 \text{ below } +x \text{ axis}$$

$$\vec{A} - \vec{B} = (53.67 \hat{i} - 1.31 \hat{j}) \text{ units}$$

notice that $\vec{A} - \vec{B} = -(\vec{B} - \vec{A})$; same magnitude but direction is different.

17

17

Name _____

Problem 3: (33 Points)

The position of a ball rolling in a straight line is given by $x = 2.0 - 3.6t + 1.1t^2$, where x is in meters and t in seconds.

(a) Determine the position of the ball at $t = 1.0$ s, and 3.0 s.

$$x(t = 1.0\text{s}) = 2.0 - 3.6(1.0\text{s}) + 1.1(1.0\text{s})^2 = -0.5\text{m}$$

$$x(t = 3.0\text{s}) = 2.0 - 3.6(3.0\text{s}) + 1.1(3.0\text{s})^2 = 1.1\text{m}$$

11

(b) What is the average velocity over the interval $t = 1.0$ s to $t = 3.0$ s

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \frac{x(3.0\text{s}) - x(1.0\text{s})}{3.0\text{s} - 1.0\text{s}} = \frac{1.1\text{m} - (-0.5\text{m})}{2.0\text{s}} = 0.8\text{m/s}$$

11

(c) What is its instantaneous velocity at $t = 2.0$ s and at $t = 3.0$ s

$$v_{\text{inst}} = \frac{dx}{dt} = -3.6 + 2.2t$$

$$v_{\text{inst}}(2.0\text{s}) = -3.6 + 2.2(2.0\text{s}) = 0.8\text{m/s}$$

$$v_{\text{inst}}(3.0\text{s}) = -3.6 + 2.2(3.0\text{s}) = 3.0\text{m/s}$$

11